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By

Dao-Zhi Zeng

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Kagawa University

Takamatsu, Kagawa 760-8523

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Spatial Economy with Multi-Manufacturing Industries*

Dao-Zhi Zeng†
Faculty of Economics, Kagawa University

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Abstract

In this paper, an economic geography model with multiple manufacturing industries is examined. In this model, the production of each manufacturing industry needs specific types of skilled and some unskilled workers. Skilled workers are mobile between regions but not across industries. The results first show that the interaction of demand, increasing returns, and transportation costs is enough to derive a separating equilibrium, in which at least two industries agglomerate in different regions. In addition, we demonstrate how the re-dispersion process observed in the literature when transport costs are small is different from the dispersion process when transport costs are high. Specifically, there is at most one industry that disperses in the re-dispersion process while all industries disperse in the dispersion process. Finally, similar results are shown to hold from the viewpoint of optimal welfare.

Key words: dispersion, multiple industries, re-dispersion, geographic concentration

JEL classification: R12, R13, F12

1 Introduction

The agglomeration of activities in a few locations is well explained by the core-periphery model of Krugman ([12]), which is a pioneering study in the field of new economic geography. As illustrated in Fujita, Krugman and Venables [8] and Fujita and Thisse [9], a typical model assumes one manufacturing sector with increasing returns to scale in its production and one agricultural sector with constant returns. There are two kinds of workers, skilled and unskilled. Skilled workers are mobile between two regions, while unskilled workers are not. Krugman [12] and Fujita et al. [8] use a Chamberlinian model of monopolistic competition developed by Dixit and Stiglitz [3], and their model turns out to be astoundingly difficult to work with, making numerical simulations necessary for most results. In contrast, Ottaviano, Tabuchi and Thisse [17] restructure the model by assuming a quasi-linear utility function with a quadratic sub-utility and the same transport cost for all varieties. Their framework makes the core-periphery model analytically solvable in the sense that it is possible to derive closed form solutions for the endogenous variables to assess analytically the number of equilibria as well as their (asymptotic) stability. Picard and Zeng [18] further develop the model to include agricultural sector and give

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†E-mail: zeng@ec.kagawa-u.ac.jp. Tel&Fax: +81-87-832-1905.

an analytical treatment to the case of positive transport costs for both manufacturing goods and agricultural goods.

The assumption of a single industry in the manufacturing sector makes the mathematical analysis easy, but it oversimplifies the reality. Fujita, Krugman and Mori [7] first analyze an economy that contains multiple groups of manufacturing goods. There, they assume a continuous space and find that a Christaller-type hierarchical urban system emerges in a self-organizing manner when an economy expands gradually based on increases in population. However, they assume that the transport costs of all goods as constant, and, therefore, their results do not provide the answers for how regions or cities evolve when transport costs decrease. There are many ways to model different industries in the manufacturing sector. For simplicity, in this paper, we focuses on the number of unskilled workers in each industry. For example, the automotive and electric goods industries are similar in the sense that they are high-tech industries that do not require a large number of unskilled workers. Both of them are quite different from the textile industry, in which many unskilled workers are involved in production. Some statistical evidence of high levels of concentration of multiple industries across the U.S. can be found in Chapter 2 of Krugman [14] and in Kim [11] and Ellison and Glaeser [5, 6]. While it is true that random chance may cause agglomeration of industries, Ellison and Glaeser [5] find that the actual pattern of US plant location is considerably more agglomerated than chance alone would explain. Therefore, it is important to clarify the internal mechanism from the viewpoint of new economic geography. Some attempts at clarification can be found in the literature. For example, Part IV of Fujita et al. [8] establishes an international trade model, which is general enough to include multiple industries and multiple countries. However, their model does not distinguish between skilled and unskilled workers, and all of them are immobile among countries. Recently, Tabuchi and Thisse [21] established a model of two industries, in which workers are mobile between regions and industries. However, a two-industry model is too simple to explain a phenomenon that is typical of multi-industries, namely, that they are geographically concentrated. In the real world, various industries agglomerate in different cities. For example, Detroit emerged as the automotive center, Seattle as the aircraft center (Boeing), Rochester as the photographic equipment center (Kodak), New York as the garment center, and Grand Rapids as the furniture center. The terms *geographic concentration* or *separating equilibrium* are used in this paper to indicate a situation in which at least two industries agglomerate in different regions. On the geographic concentration of industries in the U.S., Ellison and Glaeser [5, 6] argue that this concentration may arise for two reasons. The first reason is the existence of increasing-returns technologies and economies of scale. The second reason is the existence of natural cost advantages that are due to differences in the actual physical geography. There are some good models (e.g., Henderson [10]) that explain the importance of economies of scale and the "first nature" of a particular region; however, the reason for increasing-returns technologies is not made particularly clear in the literature. Although Krugman [12] provides a picture of the whole manufacturing sector of all industries, his model says nothing about each individual industry.

Our primary purpose in this paper is to provide a new explanation for geographic concentration by extending the single-industry model directly to the case of multiple industries. Specifically, we show in this paper that the interaction of demand, increasing returns, and transportation costs is sufficient to drive a separating equilibrium. To ease mathematical analysis, all the existing models dealing with plural industries, such as those by Tabuchi and Thisse [21] and Part IV of Fujita et al. [8], disregard the agricultural sector and assume that all workers are skilled. However, we believe that the interaction between two sectors is important when analyzing economic geography. As an example of the interaction, Krugman ([14], p.13) cites

McCarty [16] to explain a development in Green Bay, WI. He writes, "Outside the manufacturing belt, cities exist to serve the farms; inside, farms exist to serve the cities." Empirically, in China, many agricultural workers have move to cities to find jobs in manufacturing over the last two decades. Consequently, while dealing with multiple industries, the paper uses a model which is good enough to include the agricultural sector. In the model, specific types of skilled workers and some unskilled workers are required by each manufacturing industry. Skilled workers are mobile between regions but not between industries. The mobility between regions can be justified by the fact that many countries are open to skilled workers (e.g., Carlos Ghosn, the president and CEO of Nissan Motor Co., Ltd. of Japan was born in Brazil and raised as a French citizen). The immobility across industries is due to the fact that it requires some special training to become a skilled worker in an industry, and different skills are required for different industries. For example, it is rare, if not impossible, for an expert in the textile industry to become an aeronautical engineer. A manufacturing industry is also distinguished from others by its requirement for a specific number of unskilled workers. Unskilled workers within a region can worker in either agricultural, or manufacturing sector in the same region, where they would earn the same wage. Unskilled workers are not at liberty to move across regions. This is justified by the fact that, in the real world, the mobility of an unskilled worker is much lower than that of a skilled worker. We will demonstrate that there is a stable separating equilibrium when transport costs are small, a situation in which at most one industry disperses and other industries agglomerate in different regions. This is consistent with the fact observed in Tabuchi and Thisse [21], namely, no equilibrium in which two industries disperse is stable.

Our secondary purpose is to clarify the difference between the dispersion and the re-dispersion observed in the literature. When transport costs are high, the traditional model of Krugman [12] shows that the dispersing equilibrium (in which a single manufacturing industry disperses equally in two symmetric regions) is stable. When transport costs decrease, the dispersing equilibrium becomes unstable, while the agglomerating equilibrium (in which a single manufacturing industry agglomerates in one region) becomes stable. Including urban costs in the model, Tabuchi [20] shows that the dispersing equilibrium stabilizes again if the transport cost decreases further. Such dispersion is called re-dispersion and is also observed either when unskilled workers are involved in manufacturing, or when the transport costs of agricultural goods are positive (See Puga [19], Fujita et al. [8], and Picard and Zeng [18]). Because of the assumption of a single manufacturing industry, the above mentioned authors do not see any difference between dispersion and re-dispersion. In this paper, their models are extended to multi-industry cases. The result shows that, in the dispersion process, each industry allocates equally in both regions. However, surprisingly in the re-dispersion process, at most one industry disperses, while others agglomerate in different regions. This result verifies those of Fujita et al. [7] in the sense that economies of agglomeration should be considered at the industry level. Our result is understandable if we notice that the dispersion force comes from price competition, which crucially depends on the manufacturing transport cost, but the re-dispersion force comes from immobile factors, which do not depend on the manufacturing transport cost.

There are several dispersion forces known in the literature. Here, we use the agricultural sector framework developed in Picard and Zeng [18]. We have four reasons. First, it allows us to distinguish industries by their labor intensities. As we will see in Section 3, this asymmetry among industries is essential to derive a separating equilibrium. Second, it allows us to observe the share of agricultural employment in a specific region. In a manufacturing agglomerated region, many unskilled workers are employed in the manufacturing sector, and employment in the agricultural sector is relatively low. This provides an explanation for the high cost of agricultural goods in Japan. Third, the two factors (skilled and unskilled workers) in the manufacturing sector can be viewed as another set of two factors (entrepreneurs and laborers) in the footloose-entrepreneur model of Baldwin, Forslid, Martin, Ottaviano and Robert-Nicoud [1]. In this way, our model generalizes that of the above authors, and our results have some implications in their setting. The generalization comes from the fact that we use two agricultural products and consider multiple industries. Finally, within this framework, the problem presented in this model can be thoroughly solved analytically, which permits more concrete results.

The remainder of the paper is organized as follows. The basic model is presented in Section 2, in which we assume that regions are symmetric and the transport cost of agricultural goods is zero for simplicity. Section 3 contains the main results and provides an analysis for the migration equilibrium and its stability. There, we give a necessary and sufficient condition for the stability of a stable equilibrium, which tells us that, generically, there is at least one stable equilibrium. We then we show that it is impossible to derive a separating equilibrium in a model in which both regions and industries are symmetric. In contrast, a separating equilibrium, rather than full dispersion, appears when the transport cost is small enough and there are at least three asymmetric industries. Section 4 shows that most results can be directly extended to the case of asymmetric regions and the economy with a positive transport cost of agricultural goods. Section 5 gives the welfare analysis, through which we find that the population distributions in equilibrium and in optimal states are very similar. Finally, Section 6 is the conclusion.

2 The model

Our framework is traditional. There are two regions and two sectors. The first sector includes firms that produce at constant returns to scale and sell their products in perfectly competitive markets. In reference to the literature on economic geography, this sector is called agriculture and is denoted by the subscript a. The second sector includes firms producing under increasing returns to scale and competing under monopolistic competition. This is called the manufacturing sector and is denoted by the subscript m. A typical model in the literature assumes that there is only one industry in the manufacturing sector, which includes all the productions of computers, automobiles, and textiles. This makes mathematical analysis easy, but it oversimplifies the reality. In fact, the production of automobiles and computers requires different kinds of skilled workers, and more unskilled workers are necessary in textile production than in automobile production. Our model differentiates one type of production from another and supposes that there are $K \geq 3$ industries in the manufacturing sector and each industry i produces an infinite amount of differentiated varieties $x_i \in [0, N]$, which are each produced by an indivisible firm x_i. For convenience, we suppose that N is the same for all industries.

Regions are called north and south and are denoted by n and s, respectively. Temporarily, the regions are supposed to be symmetric in the sense that they are endowed with the same number (A) of unskilled workers. Therefore, there is no "first nature" location advantage in the model. We will lift this assumption in Section 4. There are K types of skilled workers, who work in specific industries within the manufacturing sector. For convenience, we suppose that the number of each type of skilled workers is the same N. Technology in industry i requires only one skilled worker of type i and ϕ_i number of unskilled workers in order to produce any amount of a variety; that is, the marginal cost of production of a variety is set equal to zero.[1] We suppose that skilled workers cannot migrate across industries. Basically, this is because it requires some special training to become a skilled worker in a specific industry, and different skills are required

[1]The results here can be generalized to the case of positive marginal cost, but the notations will be much heavier. Therefore, we retain this assumption, as Ottaviano et al. [17] did.

for different industries. Therefore, changing a profession is very costly for a skilled worker. Second, this assumption naturally extends the immobility assumption between the agricultural sector and the (whole) manufacturing sector imposed in most models in the literature. Third, the mathematical analysis in this paper is true for all $K \geq 3$. When K is as small as three, our model is close to the traditional model. The classification of industries is gross so that the immobility assumption is more acceptable. On the other hand, manufacturing production can locate in any region, and the skilled workers are mobile between regions. We denote the number of firms of type i in region n by $\lambda_i N$ and the number of firms of type i in region s becomes $(1-\lambda_i)N$. By contrast, the agricultural sector produces two agricultural varieties, each variety being grown in a single region only (for example, rice in Asia and potatoes in Europe, or Japanese rice and Thai rice). We denote agricultural varieties by the regional labels. That is, an agricultural variety n is grown in region n and variety s is grown in in region s. This assumption can be justified by Ricardian advantages in agriculture. Each unskilled worker can either work in the regional agricultural production or in an industrial firm in the residential region, so their wages are the same. Unskilled workers are not allowed to migrate between regions, and the agricultural wages in the two regions may be different.

Specifically, the preferences of a representative resident in either region are given by the following utility function over K kinds of industry goods represented by q_m, 2 kinds of agricultural goods represented by q_a and a numéraire represented by q^0:

$$\begin{aligned}U^r(q_m, q_a, q^0) =& \alpha_m \sum_{i=1}^{K} \int_0^N q^r_{m,i}(x)dx - \frac{\beta_m - \gamma_m}{2} \sum_{i=1}^{K} \int_0^N [q^r_{m,i}(x)]^2 dx \\ &- \frac{\gamma_m}{2} \sum_{i=1}^{K} \left[\int_0^N q^r_{m,i}(x)dx \right]^2 + \alpha_a(q_a^{nr} + q_a^{sr}) - \frac{\beta_a - \gamma_a}{2}[(q_a^{nr})^2 + (q_a^{sr})^2] \\ &- \frac{\gamma_a}{2}[q_a^{nr} + q_a^{sr}]^2 + q^0. \end{aligned} \tag{1}$$

Each individual maximizes utility (1) with budget constraint

$$\sum_{i=1}^{K} \int_0^N p^r_{m,i}(x) q^r_{m,i}(x)dx + p_a^{nr} q_a^{nr} + p_a^{sr} q_a^{sr} = y + \bar{q}_0,$$

where y is the income and $\bar{q}_0$ is the initial endowment that is supposed to be sufficiently large for the equilibrium consumption of the numéraire to be positive.

Let

$$\begin{aligned} a_m =& \frac{\alpha_m}{\beta_m + (N-1)\gamma_m}, \quad b_m = \frac{1}{\beta_m + (N-1)\gamma_m}, \quad c_m = \frac{\gamma_m}{(\beta_m - \gamma_m)[\beta_m + (N-1)\gamma_m]}, \\ a_a =& \frac{\alpha_a}{\beta_a + \gamma_a}, \quad b_a = \frac{1}{\beta_a + \gamma_a}, \quad c_a = \frac{\gamma_a}{(\beta_a - \gamma_a)(\beta_a + \gamma_a)}. \end{aligned}$$

The demand functions for manufacturing goods are then

$$\begin{aligned} q^{nn}_{m,i} =& a_m - (b_m + Nc_m)p^{nn}_{m,i} + c_m P^n_{m,i}, \quad q^{ns}_{m,i} = a_m - (b_m + Nc_m)p^{ns}_{m,i} + c_m P^s_{m,i}, \\ q^{ss}_{m,i} =& a_m - (b_m + Nc_m)p^{ss}_{m,i} + c_m P^s_{m,i}, \quad q^{sn}_{m,i} = a_m - (b_m + Nc_m)p^{sn}_{m,i} + c_m P^n_{m,i}, \end{aligned}$$

where the price indices of manufacturing products are

$$P^n_{m,i} = \lambda_i N p^{nn}_{m,i} + (1-\lambda_i)N p^{sn}_{m,i}, \quad P^s_{m,i} = (1-\lambda_i)N p^{ss}_{m,i} + \lambda_i N p^{ns}_{m,i}.$$

We assume that N is large enough so that a firm's price does not change the price index, which is the standard monopolistic competition assumption. From the FOC of maximizing the profits, we obtain

$$p_{m,i}^{nn} = \frac{a_m + c_m P_{m,i}^{n}}{2(b_m + c_m N)}, \quad p_{m,i}^{sn} = p_{m,i}^{nn} + \frac{\tau}{2},$$
$$p_{m,i}^{ss} = \frac{a_m + c_m P_{m,i}^{s}}{2(b_m + c_m N)}, \quad p_{m,i}^{ns} = p_{m,i}^{ss} + \frac{\tau}{2}.$$

Solving them, we obtain

$$p_{m,i}^{nn} = \frac{2a_m + \tau c_m (1-\lambda_i) N}{2(2b_m + c_m N)}, \quad p_{m,i}^{sn} = p_{m,i}^{nn} + \frac{\tau}{2},$$
$$p_{m,i}^{ss} = \frac{2a_m + \tau c_m \lambda_i N}{2(2b_m + c_m N)}, \quad p_{m,i}^{ns} = p_{m,i}^{ss} + \frac{\tau}{2}.$$

Therefore,

$$\begin{aligned}
q_{m,i}^{nn} =& \frac{(b_m + c_m N)[2a_m + c_m N(1-\lambda_i)\tau]}{2(2b_m + c_m N)}, \\
q_{m,i}^{ns} =& \frac{(b_m + c_m N)\{2a_m - [2b_m + c_m N(1-\lambda_i)]\tau\}}{2(2b_m + c_m N)}, \\
q_{m,i}^{ss} =& \frac{(b_m + c_m N)(2a_m + c_m N\lambda_i \tau)}{2(2b_m + c_m N)}, \\
q_{m,i}^{sn} =& \frac{(b_m + c_m N)[2a_m - (2b_m + c_m N\lambda_i)\tau]}{2(2b_m + c_m N)}.
\end{aligned}$$

For agricultural products, the demand functions are

$$q_a^{nn} = a_a - b_a p_a^{nn} + c_a(p_a^{sn} - p_a^{nn}), \quad q_a^{ns} = a_a - b_a p_a^{ns} + c_a(p_a^{ss} - p_a^{ns}), \tag{2}$$
$$q_a^{ss} = a_a - b_a p_a^{ss} + c_a(p_a^{ns} - p_a^{ss}), \quad q_a^{sn} = a_a - b_a p_a^{sn} + c_a(p_a^{nn} - p_a^{sn}). \tag{3}$$

We have assumed that the agricultural production is at constant returns to scale, and the price is determined by letting the demands be equal to the supplies. For convenience, we suppose that each unskilled worker (farmer) produces one unit of agricultural product. Since there are $\sum_{i=1}^{K} \lambda_i N\phi_i$ (resp. $\sum_{i=1}^{K}(1-\lambda_i)N\phi_i)$) unskilled workers who work in manufacturing firms in region n (resp. s), we have

$$A - \sum_{i=1}^{K} \lambda_i N\phi_i = q_a^{nn}\left(A + \sum_{i=1}^{K}\lambda_i N\right) + q_a^{ns}\left[A + \sum_{i=1}^{K}(1-\lambda_i)N\right],$$
$$A - \sum_{i=1}^{K} (1-\lambda_i) N\phi_i = q_a^{sn}\left(A + \sum_{i=1}^{K}\lambda_i N\right) + q_a^{ss}\left[A + \sum_{i=1}^{K}(1-\lambda_i)N\right].$$

For the simplicity of notations, we assume that the transport cost for agricultural goods is temporarily zero. As will be shown in Section 4, all the main results here can be generalized to the case of positive agricultural transport cost. Then, by combining (2) and (3), we obtain

$$p_a^{ns} = p_a^{nn} = \frac{a_a}{b_a} + \frac{\sum_{i=1}^{K}(c_a + b_a\lambda_i)N\phi_i - 2c_a A - b_a A}{b_a(b_a + 2c_a)(2A + L)},$$

$$p_a^{sn} = p_a^{ss} = \frac{a_a}{b_a} + \frac{\sum_{i=1}^{K}[c_a + b_a(1-\lambda_i)]N\phi_i - 2c_aA - b_aA}{b_a(b_a+2c_a)(2A+L)}.$$

After determination of product prices, we know that the gross profits of a manufacturing firm in industry i and region l for selling products in region k Π_i^{lk} are

$$\begin{aligned}
\Pi_i^{nn} =&(A + \sum_{i=1}^{K} \lambda_i N)(b_m + c_m N)(p_{m,i}^{nn})^2, \\
\Pi_i^{ns} =&[A + \sum_{i=1}^{K}(1-\lambda_i)N](b_m + c_m N)(p_{m,i}^{ns} - \tau)^2, \\
\Pi_i^{ss} =&[A + \sum_{i=1}^{K}(1-\lambda_i)N](b_m + c_m N)(p_{m,i}^{ss})^2, \\
\Pi_i^{sn} =&(A + \sum_{i=1}^{K} \lambda_i N)(b_m + c_m N)(p_{m,i}^{sn} - \tau)^2.
\end{aligned}$$

By the assumption of free entry, the profits of firms should be zero, and we have

$$\begin{aligned}
w_i^n + \phi_i p_a^{nn} &= \Pi_i^{nn} + \Pi_i^{ns} \quad \text{if } \lambda_i > 0, \\
w_i^s + \phi_i p_a^{ss} &= \Pi_i^{sn} + \Pi_i^{ss} \quad \text{if } \lambda_i < 1.
\end{aligned}$$

Therefore, in an inner equilibrium, the wage differential is

$$w_i^n - w_i^s = \Pi_i^{nn} + \Pi_i^{ns} - \phi_i p_a^{nn} - \Pi_i^{sn} - \Pi_i^{ss} + \phi_i p_a^{ss}.$$

The indirect utility function for residents in region r is

$$\begin{aligned}
V^r(q_m, q_a, q^0) =& \sum_{i=1}^{K} \alpha_m \int_0^N q_{m,i}^r(x)dx - \frac{\beta_m - \gamma_m}{2} \sum_{i=1}^{K} \int_0^N [q_{m,i}^r(x)]^2 dx \\
&- \frac{\gamma_m}{2} \sum_{i=1}^{K} \left[\int_0^N q_{m,i}^r(x)dx \right]^2 + \alpha_a(q_a^n + q_a^s) - \frac{\beta_a - \gamma_a}{2}[(q_a^n)^2 + (q_a^s)^2] \\
&- \frac{\gamma_a}{2}[q_a^n + q_a^s]^2 + q^0 \\
=& \frac{(a_m)^2 L}{2b_m} - a_m \sum_{i=1}^{K} \int_0^N p_{m,i}^r(x)dx + \frac{b_m + c_m N}{2} \sum_{i=1}^{K} \int_0^N [p_{m,i}^r(x)]^2 dx \\
&- \frac{c_m}{2} \sum_{i=1}^{K} \left(\int_0^N p_{m,i}^r(x)dx \right)^2 + \frac{(a_a)^2}{b_a} - a_a(q_a^n + q_a^s) \\
&+ \frac{b_a + 2c_a}{2}[(q_a^n)^2 + (q_a^s)^2] - \frac{c_a}{2}[q_a^n + q_a^s]^2 + y + \bar{q}_0 \\
=& S_m^r + S_a^r + w^r + \bar{q}_0,
\end{aligned}$$

where w^r is the wage income and ($\bar{r}$ is the other region), S_m^r and S_a^r are the consumer surpluses in region r of the manufacturing goods and the agricultural goods, respectively:

$$S_m^r = \frac{(a_m)^2 L}{2b_m} - a_m \sum_{i=1}^{K} \int_0^N p_{m,i}^r(x)dx + \frac{b_m + c_m N}{2} \sum_{i=1}^{K} \int_0^N [p_{m,i}^r(x)]^2 dx$$

$$
-\frac{c_m}{2}\sum_{i=1}^{K}\left(\int_0^N p_{m,i}^r(x)dx\right)^2,
$$

$$
S_a^r = \frac{a_a^2}{b_a} - a_a[p_a^{rr} + p_a^{\bar{r}r}] + \frac{b_a + 2c_a}{2}[(p_a^{rr})^2 + (p_a^{\bar{r}r})^2] - \frac{c_a}{2}[p_a^{rr} + p_a^{\bar{r}r}]^2.
$$

3 Equilibrium

Before the essential analysis, we add the following notations, where $i, j = 1, \dots, K, (i \neq j)$:

$$
\nu = \frac{(b_m + c_m N)}{2(2b_m + c_m N)^2}[6b_m^2 + c_m^2 N(2A + L) + 2b_m c_m(2A + L + 2N)]\tau^2 - \frac{2a_m(b_m + c_m N)(3b_m + 2c_m N)}{(2b_m + c_m N)^2}\tau,
$$

$$
\mu = \frac{(b_m + c_m N)(3b_m + 2c_m N)}{(2b_m + c_m N)^2}(b_m\tau - 2a_m)\tau,
$$

$$
\delta_{ij} = \begin{cases} N\left[\nu + \frac{2\phi_i^2}{(b_a+2c_a)(2A+L)}\right] & \text{if } i = j, \\ N\left[\mu + \frac{2\phi_i\phi_j}{(b_a+2c_a)(2A+L)}\right] & \text{if } i \neq j. \end{cases}
$$

These notations will be explained after (7). Since $\delta_{ij} = \delta_{ji}$, matrix $\Delta_k = (\delta_{ij})_{k\times k}$ $(k = 1, \dots, K)$ is symmetric. Note that μ and ν do not depend on the parameters of the agricultural sector, and

$$
\nu - \mu = \frac{c_m(2A + L)(b_m + c_m N)\tau^2}{2(2b_m + c_m N)} > 0. \tag{4}
$$

For the sake of conciseness, we focus on the case in which all varieties in both sectors are consumed and their prices are positive.[2] In notation, we require $q_{m,i}^{lk} > 0$, $p_{m,i}^{lk} > 0$, $q_a^{lk} > 0$ and p_a^{lk} hold for all $\boldsymbol{\lambda}$ and any regions l, k, and industry i. These conditions are satisfied if the transportation cost is sufficiently small, that is

$$
\tau < \frac{2a_m}{2b_m + c_m N} \equiv \tau_{\text{trade}}, \tag{5}
$$

$$
\sum_{i=1}^{K}\phi_i N < A < a_a(2A + L) + \frac{c_a}{b_a + 2c_a}\sum_{i=1}^{K}\phi_i N. \tag{6}
$$

Notation τ_{trade} is exactly the same as in Ottaviano et al. [17], and (5) is also assumed in their model. The first inequality of (6) says that the population of unskilled workers in each region is large enough to serve the whole manufacturing sector. In other words, the labor supply of unskilled workers is never depleted in each region. The agglomeration of the manufacturing sector is impossible without this inequality. The second inequality of (6) holds automatically for $a_a \geq 1/2$, which is true when the intensity α_a of preference for agricultural goods is high enough. We take (5) and (6) as assumptions in the sequel. As an immediately consequence, (5) implies that $\tau < 2a_m/b_m$ and hence $\mu < 0$ holds for all $\tau > 0$.

[2] We do not mean that the contrary case is not interesting. In fact, Behrens [2] considers the case of corner equilibria for large τ and find that agglomeration may occur even when there is not trade.

The utility differential between two regions is

$$
\begin{aligned}
&V_i^n(\boldsymbol{\lambda}) - V_i^s(\boldsymbol{\lambda})\\
&\quad = S_n^m - S_s^m + S_n^a - S_s^a + w_n^i - w_s^i\\
&\quad = (\frac{1}{2} - \lambda_i)N\Big\{\frac{(b_m + c_m N)}{2(2b_m + c_m N)^2}[6b_m^2 + c_m^2 N(2A + L) + 2b_m c_m[2A + L + 2N]\tau^2\\
&\qquad\qquad - \frac{2a_m(3b_m + 2c_m N)(b_m + c_m N)}{(2b_m + c_m N)^2}\tau + \frac{2(\phi_i)^2}{(b_a + 2c_a)(2A + L)}\Big\}\\
&\qquad + \sum_{j=1, j\neq i}^{K}(\frac{1}{2} - \lambda_j)N\Big[\frac{(b_m + c_m N)(3b_m + 2c_m N)}{(2b_m + c_m N)^2}(b_m\tau - 2a_m)\tau + \frac{2\phi_i\phi_j}{(b_a + 2c_a)(2A + L)}\Big]\\
&\quad = \sum_{j=1}^{K}(\frac{1}{2} - \lambda_j)\delta_{ij}. \qquad (7)
\end{aligned}
$$

The above expression provides an economic explanation of δ_{ij}:

$$\delta_{ij} = -\frac{\partial[V_i^n(\boldsymbol{\lambda}) - V_i^s(\boldsymbol{\lambda})]}{\partial\lambda_j}.$$

It is the impact on the differential of type i's utility levels between two regions if one skilled worker of type j moves from region n to region s. The impact is from both the manufacturing sector and the agricultural sector, which are summarized by $N\mu$ (or $N\nu$ when $i = j$) and $2N\phi_i\phi_j/[(b_a + 2c_a)(2A + L)]$, respectively. In the manufacturing sector, when a skilled worker of type j moves from region n to region j, the price index of industry j in region n changes. Furthermore, because of the immobility of skilled workers across industries, a firm in industry j also moves following the skilled worker who has moved. This explains (4), which tells us that this effect on type j industry from such a move is larger than the effect on other types. Because the skilled workers are mobile between regions, the impact from the manufacturing sector is small when τ converges to zero. In contrast, the impact from the agricultural sector is independent of τ.

Since we assume that the skilled workers are immobile across industries, the impact from the manufacturing sector is bigger on the same industry than on others, as described by (4).

In order to study the stability of a spatial equilibrium, following established tradition in economic geography, we assumed that markets for goods adjust instantaneously, while interregional migration of firms and workers is relatively slow, implying that the wages adjust much faster than the labor share. The dynamics of labor shares is as follows:[3]

$$\frac{d\lambda_i}{dt} = V_i^n(\boldsymbol{\lambda}) - V_i^s(\boldsymbol{\lambda}) = \sum_{j=1}^{K}(\frac{1}{2} - \lambda_j)\delta_{ij}. \qquad (8)$$

If $|\Delta_K| \neq 0$, then $(1/2, \ldots, 1/2)$ is the unique inner equilibrium of (8). Since Δ_K is symmet-

[3] Another dynamics, called replicator dynamics, is also widely applied in the literature. However, their equilibria and their stability conditions turn out to be the same (see Tabuchi and Zeng [23]). This paper chooses (8) because it looks simpler.

ric, this inner equilibrium is stable if and only if all the principal minors are positive:[4]

$$|\Delta_1| \equiv \delta_{11} > 0, \quad |\Delta_2| \equiv \begin{vmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{vmatrix} > 0, \quad \ldots, \quad |\Delta_K| \equiv \begin{vmatrix} \delta_{11} & \cdots & \delta_{1K} \\ \vdots & \ddots & \vdots \\ \delta_{K1} & \cdots & \delta_{KK} \end{vmatrix} > 0. \tag{9}$$

As shown in Appendix A, we have an explicit expression for $|\Delta_k|$, $k = 1, \ldots, K$:

$$\begin{aligned} |\Delta_k| =& N^k(\nu-\mu)^{k-2}\Bigg\{(\nu-\mu)[\nu+(k-1)\mu] + \frac{2(\nu-\mu)}{(b_a+2c_a)(2A+L)}\sum_{i=1}^{k}\phi_i^2 \\ &+ \frac{\mu}{(b_a+2c_a)(2A+L)}\sum_{i=1}^{k}\sum_{j=1}^{k}(\phi_i-\phi_j)^2\Bigg\}. \end{aligned} \tag{10}$$

The above expression allows us to give an analytical argument on the stability of an equilibrium, which will provide us a whole picture of the evolution process when the transport cost τ decreases.

On the other hand, we define the following potential function

$$\mathcal{P}(\boldsymbol{\lambda}) = -\frac{1}{2}\sum_{i=1}^{K}\sum_{l=1}^{K}\delta_{il}(\frac{1}{2}-\lambda_i)(\frac{1}{2}-\lambda_l). \tag{11}$$

Then, conditions (9) tell us that distribution $\boldsymbol{\lambda}^* = (\lambda_1^*, \ldots, \lambda_K^*)$ is the equilibrium of (8) if and only if it is an extreme point of $\mathcal{P}(\boldsymbol{\lambda})$, and $\boldsymbol{\lambda}^*$ is a stable equilibrium of (8) if and only if it is a locally unique maximal point[5] of $\mathcal{P}(\boldsymbol{\lambda})$.

In Appendix B, we show:

Theorem 1 *Generically, there is a stable equilibrium of (8).*

In the following, we examine the stability conditions (9) and its economic implications in detail.

3.1 No unskilled labor in manufacturing

This simplest case extends the model of Ottaviano et al. [17] for more than one manufacturing industry, in which no unskilled workers are needed in manufacturing productions. That is, $\phi_i = 0$ for each $i = 1, \ldots, K$. From (10), it holds that

$$|\Delta_k| = N^k(\nu-\mu)^{k-1}[\nu+(k-1)\mu].$$

Since $\mu < 0$ and $\nu - \mu > 0$, the above expression is positive for all $k = 1, \ldots, K$ if and only if $\nu + (K-1)\mu > 0$, or equivalently,

$$\tau > \frac{4a_m(3b_m+2c_mN)}{c_m(\frac{2A}{K}+N)(2b_m+c_mN)+2b_m(3b_m+2c_mN)} \equiv \tau_0 \tag{12}$$

holds. If the inverse inequality of (12) holds, then full dispersion is unstable. However, we have

$$V_i^n(\mathbf{1}) - V_i^s(\mathbf{1}) = -\frac{N[\nu+(K-1)\mu]}{2} > 0;$$

[4]Strictly speaking, we do not know the exact stability when one of the inequalities in (9) becomes equality. Here we omit this case because it happens with only a finite number of τ values according to (10).

[5]By this we mean that there is a neighborhood of $\boldsymbol{\lambda}$ in which $\boldsymbol{\lambda}$ is the only maximizer.

therefore, full agglomeration is stable. The break point and the sustain point are equal to τ_0, which solves $\nu + (K-1)\mu = 0$.

For constant N, τ_0 of (12) is increasing in K. Therefore, for larger K and therefore a larger number of skilled workers, full agglomeration occurs in a larger range of τ. Particularly, if K is large enough so that $\tau_0 \geq \tau_{\text{trade}}$ (which is defined in (5)), then a full dispersion equilibrium does not exist and full agglomeration always appears.

Note that there is no re-dispersion process, which is observed in Ottaviano et al. [17]. The reason is that neither urban costs nor agricultural goods are considered here.

3.2 Double symmetry

As in Section 16.3 of Fujita et al. [8], this section imposes the double-symmetry assumption, assuming that both the regions and the industries are symmetric. Specifically, we let $\phi_i = \phi \geq 0$ for each $i = 1, \ldots, K$.

Fujita et al. [8] obtain a separating equilibrium for two industries, assuming that workers are immobile between regions. The immobile assumption is acceptable for international trade research but not for regional economics. In contrast, our model supposes that skilled workers are mobile. Can we obtain a separating equilibrium for some τ? The following conclusion gives a negative answer, which suggests that we need a model with either asymmetric regions or asymmetric industries to derive a separating equilibrium.

Theorem 2 (Impossibility) *No separating equilibrium is stable in a model with double symmetry.*

The proof is relegated to Appendix C. There, we have further shown that, in a model with double symmetry, there is no stable equilibrium in which some industries agglomerate in only one region, and others (at least one industry) disperse.[6] In other words, the only possible equilibria are full agglomeration and full dispersion.

In the following, we suppose that $\phi > 0$ and derive the conditions for full agglomeration and full dispersion. The expression of Δ_k is simplified as

$$\Delta_k = N^k(\nu-\mu)^{k-1}\left[\nu + (k-1)\mu + \frac{2k\phi^2}{(b_a+2c_a)(2A+L)}\right], \quad k = 1, \ldots, K. \tag{13}$$

There are two possible agglomerations, one in region n and another in region s. Since the regions are symmetric, both equilibria are stable if and only if one of them is stable, which happens if

$$V_i^n(\mathbf{1}) - V_i^s(\mathbf{1}) = -\frac{N}{2}\left[\nu + (K-1)\mu + \frac{2K\phi^2}{(b_a+2c_a)(2A+L)}\right] > 0, \quad \forall i = 1, \ldots, K \tag{14}$$

is positive for any $i = 1, \ldots, n$. Let

$$\phi_\sharp = \frac{Ka_m(3b_m+2c_mN)}{2b_m+c_mN}\sqrt{\frac{(b_m+c_mN)(b_a+2c_a)(2A+L)}{6Kb_m^2+6b_mc_mL+c_m^2LN+2Ac_m(2b_m+c_mN)}}.$$

If $\phi < \phi_\sharp$, then there are two threshold values $\tau_\sharp^1 < \tau_\sharp^2$ of transport cost which solve equation

$$\nu + (K-1)\mu + \frac{2K\phi^2}{(b_a+2c_a)(2A+L)}$$

[6] Such a stable equilibrium is obtained in Tabuchi and Thisse [21], where two industries are supposed to be asymmetric in the sense that their products are transported with different costs.

$$
\begin{aligned}
=&\frac{(b_m+c_mN)[c_m(2b_m+c_mN)(2A+L)+2b_m(3b_m+2c_mN)K]}{2(2b_m+c_mN)^2}\tau^2\\
&-\frac{2a_m(b_m+c_mN)(3b_m+2c_mN)}{(2b_m+c_mN)^2}K\tau+\frac{2K\phi^2}{(b_a+2c_a)(2A+L)}\\
=&0.
\end{aligned}
$$

We conclude the following:

Proposition 1 *If $\phi > \phi_\sharp$, then the full dispersion is always stable. If $\phi < \phi_\sharp$, then full dispersion is stable $\tau \in (0, \min\{\tau_{\text{trade}}, \tau_\sharp^1\}) \cup (\min\{\tau_{\text{trade}}, \tau_\sharp^2\}, \tau_{\text{trade}})$, and full agglomeration is stable when $\tau \in (\min\{\tau_{\text{trade}}, \tau_\sharp^1\}, \min\{\tau_{\text{trade}}, \tau_\sharp^2\})$.*

The proof is in Appendix D. This conclusion generalizes Picard and Zeng [18] directly. When ϕ is not too high, the regions go from full dispersion to full agglomeration and to full dispersion again (the so-called re-dispersion).

This fact has already been partly observed in Section 16.5 of Fujita et al. [8] based on a model of two industries. In the following, we show that Proposition 1 depends essentially on the symmetry assumption of industries.

3.3 Industries with different labor intensities

In general, the industries are different and their productions require a different number of unskilled workers. We will find that the results change dramatically from the double symmetry case. Without loss of generality, we suppose that industry 1 is the most labor-intensive industry and K the least labor-intensive one. Specifically, we let

$$\phi_1 \geq \ldots \geq \phi_K \text{ with at least one inequality holding strictly and } \phi_1 > 0. \tag{15}$$

First of all, the following result tells us that different industries locate sequentially in any stable equilibrium $\boldsymbol{\lambda}^* = (\lambda_1^*, \ldots, \lambda_K^*)$. Consequently, any industry i' such that $i < i' < j$ agglomerates in region n, if industries i and j agglomerate in region n. This theoretically verifies the simulation result in Section 15.2.1 of Fujita et al. [8], namely, that the most labor-intensive industry is the first to move out of the full agglomeration state.

Proposition 2 *For different industries $i_1 < i_2 < i_3$, then*

$$\min\{\lambda_{i_1}^*, \lambda_{i_3}^*\} \leq \lambda_{i_2}^* \leq \max\{\lambda_{i_1}^*, \lambda_{i_3}^*\} \tag{16}$$

The proof is in Appendix E. Roughly speaking, the cost of unskilled workers is an important factor to determine the location of a firm. Because of (15), if the firms of industry i_1 and i_3 are all located in region n, then the wages of unskilled workers in region n are also suitable for any firms in industry i_2 between i_1 and i_3.

3.3.1 The case of small τ

We pay attention to the case when τ is small. First, we show that, differently from the re-dispersion observed in Picard and Zeng [18], the full dispersion does not appear again when there are different ϕ_i.

Theorem 3 *For $K \geq 2$, there is no inner stable equilibrium of (8) when τ is small enough.*

The proof is in Appendix F. As stated in the theorem, the conclusion is true even if $K = 2$. Furthermore, although here we assume that two regions are symmetric in the sense that the populations of unskilled workers are the same, this will be generalized in Section 4 for an asymmetric model. Based on those facts, Proposition 6 concludes that there is no stable equilibrium in which at least two dispersing industries when τ is small.

On the other hand, Theorem 1 already tells us that, generically, there is a stable equilibrium. The above argument clarifies that the only possible stable equilibrium should be separating when τ is small enough. In other words, in a stable equilibrium, at most one industry disperses while others separately agglomerate in different regions. To determine such an equilibrium, we specify an industry i^d by conditions

$$\phi_1 + \ldots + \phi_{i^d-1} \leq \phi_{i^d} + \ldots + \phi_K, \tag{17}$$
$$\phi_1 + \ldots + \phi_{i^d} > \phi_{i^d+1} + \ldots + \phi_K. \tag{18}$$

Proposition 2 says that any industry j such that $1 < j < i^d - 1$ agglomerates in region n if industries 1 and $i^d - 1$ agglomerate in region n. Therefore, i^d is the threshold industry name such that dispersing industry i^d and separating others can equalize the manufacturing employment in both regions.[7] From (15), we know that $i^d \geq 1$ is uniquely determined and $\phi_{i^d} > 0$. Note that i^d is independent of parameters $\alpha_m, \beta_m, \gamma_m$ and $\alpha_a, \beta_a, \gamma_a$.

Proposition 3 *If*

$$\phi_{i^d-1} > \phi_{i^d}, \tag{19}$$

then the equilibrium in which industries $1, \ldots, i^d - 1$ locate in region n, industries $i^d + 1, \ldots, K$ locate in region s, and industry i^d either disperses or agglomerates in region s is stable for sufficiently small τ.

The proof is in Appendix G.

Attention will now be turned to the above conclusion.

First, it is interesting to find that the "re-dispersion" is quite different from the previous dispersion. Specifically, all industries disperse in the dispersion process, but at most one industry disperses in the re-dispersion process. In addition, since both regions have the same number of total unskilled workers, the shares of agricultural employment in two regions are the same in the dispersion process but different in the re-dispersion process. In fact, from (43) in Appendix G, we know that the ratio of skilled workers in region n of the dispersing industry is

$$\lambda^*_{i^d} = \frac{[\nu + (K + 1 - 2i^d)\mu](b_a + 2c_a)(2A + L) + \phi_{i^d}(\phi_{i^d} + \ldots + \phi_K - \phi_1 - \ldots - \phi_{i^d-1})}{\nu(b_a + 2c_a)(2A + L) + 2\phi^2_{i^d}}.$$

On the other hand, in equilibrium $(1, \ldots, 1, \lambda_{i^d}, 0, \ldots, 0)$, the numbers of unskilled workers employed in agricultural sectors of two regions are equal only if λ_{i^d} is

$$\lambda^a_{i_d} \equiv \frac{\phi_{i^d} + \ldots + \phi_K - \phi_1 - \ldots - \phi_{i^d-1}}{2\phi_{i^d}}.$$

Although $\lim_{\tau \to 0} \lambda^*_{i^d} = \lambda^a_{i_d}$, we cannot expect the equality of $\lambda^*_{i^d}$ and $\lambda^a_{i^d}$ for a positive τ. The difference between dispersion and re-dispersion is understandable because the dispersion force

[7] However, as we will see in the proof of Proposition 3, the distribution of industry i^d is determined by equalizing the wages of skilled workers of type i^d rather than by equalizing the manufacturing employment of unskilled workers in both regions.

comes from price competition, which heavily depends on transport cost τ, while the re-dispersion force mainly comes from the cost of unskilled workers, which does not depend on τ. If there is only one industry, the re-dispersion force makes the only type of skilled worker equally dispersed in two regions. However, when there is more than one industry, the re-dispersion force does not distinguish whether each of the industries is equally dispersed or the whole manufacturing sector is equally dispersed while an individual industry is agglomerated. Our model derives the latter because the price competition induces the agglomeration of individual industry, as shown in Ottaviano et al. [17].

Second, the result that different industries agglomerate in different cities shed light on the appearances of Toyota city in Japan and Silicon Valley in the U.S. Marshall [15] explained agglomeration by labor-market pooling, supply of intermediate goods, and knowledge spillovers. Now we can see that the interaction of demand, increasing returns, and transportation costs is enough to derive a separating equilibrium.

Third, the industries are separated in order of their labor intensities. This is because the wages of unskilled workers are important in determining the location of industries. The industries requiring more unskilled workers look for a region with cheaper unskilled workers, so they arrive in the same region and push other industries requiring fewer unskilled workers to the other region. The resulting distribution of industries is similar to but different from that of the Ricardian model. The Ricardian model with many goods (Dornbusch, Fischer and Samuelson [4]) shows that trade is mutually beneficial to both regions if the productions in both regions are specialized according to the (relatively) comparative advantage of industries. Two results are similar because the regions are specialized in the productions in both models. However, the results are different because our model supposes that both regions are symmetric and there is no comparative advantage between them. Regions are specialized according to their *absolute* labor intensities.

Finally, we find that when τ becomes smaller, the number of dispersing industries decreases to one. This is supported by the empirical result of Ellison and Glaeser [5], namely, that almost all industries are somewhat localized.

Condition (19) is essential in the proof. If $\phi_{i^d-1} = \phi_{i^d}$, then, for $j = i^d - 1$, the second item of (47) disappears and the first term of (47) is positive for sufficiently small τ because $\phi_{i^d-1} > 0$. Therefore, the reverse inequality of (46) holds for $j = i^d - 1$, and the separating equilibrium becomes unstable. This is consistent with Theorem 2, saying that we cannot expect separating equilibrium in a framework with double symmetry.

We now show that a stable separating equilibrium can be observed only when the number of industries $K \geq 3$. In other words, both distributions $(1,0)$ and $(0,1)$ are unstable when $K = 2$. In fact, the distribution $(1,0)$ is stable only if

$$0 < V_1^n(1,0) - V_1^s(1,0) = -\frac{1}{2}\delta_{11} + \frac{1}{2}\delta_{12},$$
$$0 > V_2^n(1,0) - V_2^s(1,0) = -\frac{1}{2}\delta_{21} + \frac{1}{2}\delta_{22}.$$

They are equivalent to

$$\nu - \mu + \frac{2\phi_1(\phi_1 - \phi_2)}{(b_a + 2c_a)(2A + L)} < 0 \text{ and } \mu - \nu + \frac{2\phi_2(\phi_1 - \phi_2)}{(b_a + 2c_a)(2A + L)} < 0,$$

respectively. Adding them up obtains

$$\frac{2(\phi_1^2 - \phi_2^2)}{(b_a + 2c_a)(2A + L)} < 0,$$

which contradicts $\phi_1 > \phi_2$. A similar argument shows that distribution $(0, 1)$ is also unstable. This explains why no separating equilibrium occurs in the two-industry model established in Tabuchi and Thisse [21].

3.3.2 The case of large τ

Full dispersion is possible for large τ. A dispersion equilibrium $(1/2, \dots, 1/2)$ is stable if and only if Δ_k is positive for all $k = 1, \dots, K$. Comparing with the case of symmetric industries, the last term of (10) emerges, which makes it more difficult for Δ_k to be positive. However, as shown in the example of Section 3.3.4, full dispersion often occurs for large τ.

3.3.3 The case of intermediate τ

This case is very rich and complicated. The number of dispersing industries is indeterminate and there are many equilibrium patterns.

Typically, we may have full agglomeration.

Proposition 4 *Full agglomeration equilibria* $\mathbf{1} = (1, \dots, 1)$ *and* $\mathbf{0} = (0, \dots, 0)$ *are stable if and only if*

$$\sum_{j=1}^{K} \delta_{1j} = \nu + (K-1)\mu + \frac{2\phi_1}{(b_a + 2c_a)(2A + L)} \sum_{j=1}^{K} \phi_j < 0. \tag{20}$$

Proof: From (7), we have $V_i^n(\mathbf{1}) - V_i^s(\mathbf{1}) = (-1/2)\sum_{j=1}^{K} \delta_{ij} > 0$ for all $i = 1, \dots, K$. Due to (15), the equilibrium $\mathbf{1}$ is stable if and only if (20) holds. The case of equilibrium $\mathbf{0}$ is similar. □

Two comments follow. First, using primitive parameters, (20) is equivalent to

$$\phi_1 \sum_{j=1}^{K} \phi_j < \frac{K^2 a_m^2 (3b_m + 2c_m N)^2 (b_m + c_m N)(b_a + 2c_a)(2A + L)}{(2b_m + c_m N)^2 [6Kb_m^2 + 6b_m c_m L + c_m^2 LN + 2Ac_m(2b_m + c_m N)]}$$

and $\tau \in (\min\{\tau_{\text{trade}}, \tau_{\sharp\sharp}^1\}, \min\{\tau_{\text{trade}}, \tau_{\sharp\sharp}^2\})$, where $\tau_{\sharp\sharp}^1 < \tau_{\sharp\sharp}^2$ are solutions of

$$\nu + (K-1)\mu + \frac{2\phi_1}{(b_a + 2c_a)(2A + L)} \sum_{j=1}^{K} \phi_j = 0.$$

Therefore, (20) is true for an intermediate τ value. Second, since

$$\delta_{11} - \delta_{1j} = (\nu - \mu) + \frac{2\phi_1(\phi_1 - \phi_j)}{(b_a + 2c_a)(2A + L)} > 0$$

for $j = 2, \dots, K$, (20) holds if $\delta_{11} \le 0$.

To illustrate other possible equilibria, we consider the case of $K = 3$ below.

Proposition 5 (i). *Separating equilibria* $(0, 1, 1)$ *and* $(1, 0, 0)$ *are stable if and only if*

$$\delta_{12} + \delta_{13} - \delta_{11} > 0 \quad \text{and} \quad \delta_{21} - \delta_{22} - \delta_{23} > 0. \tag{21}$$

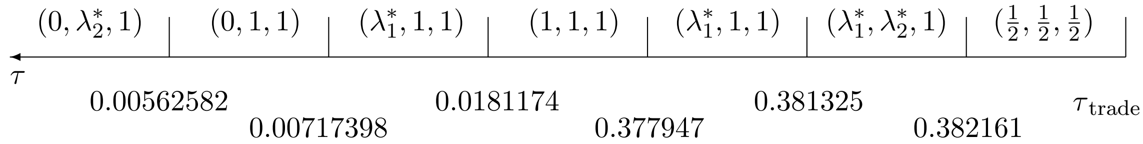

Figure 1: The whole evolution process

(ii). *There is a stable equilibrium in which only industry 1 disperses if and only if*

$$\delta_{12} + \delta_{13} - \delta_{11} < 0 < \delta_{11} + \delta_{12} + \delta_{13}, \tag{22}$$

$$\delta_{11}(\delta_{21} + \delta_{22} + \delta_{23}) < \delta_{21}(\delta_{11} + \delta_{12} + \delta_{13}). \tag{23}$$

(iii) *There is a stable equilibrium in which industries 1 and 2 disperse if and only if*

$$\delta_{11} > 0, \Delta_2 > 0, \Delta_3 < 0, \tag{24}$$

$$\delta_{11}(\delta_{21} + \delta_{22} + \delta_{23}) > \delta_{21}(\delta_{11} + \delta_{12} + \delta_{13}), \tag{25}$$

$$-\Delta_2 < \begin{vmatrix} \delta_{12} & \delta_{13} \\ \delta_{22} & \delta_{23} \end{vmatrix} < \Delta_2. \tag{26}$$

The proof is in Appendix H.

It is not surprising that the first inequality of (21) is complementary to the first inequality of (22), the second inequality of (22) is complementary to that of (20), and that of (25) is complementary to that of (23).

3.3.4 The whole process

Here, we illustrate the whole evolution picture for decreasing τ by an example using three industries as an example. The parameters are specified as follows:

$$K = 3, A = 40, N = 1, a_m = 10, b_m = 15, c_m = 4,$$
$$a_a = 3, b_a = 4, c_a = 5, \phi_1 = 15, \phi_2 = 11, \phi_3 = 9.$$

These parameters satisfy (6). The τ_{trade} of (5) turns out to be 0.588235. Similar to the situation of a single industry, a full dispersing equilibrium $(1/2, 1/2, 1/2)$ is stable for large τ (exactly $0.382161 < \tau < \tau_{\text{trade}}$). This is because of the need to serve final consumers at high trade costs. As soon as this full dispersing equilibrium becomes unstable, a type $(\lambda_1^*, \lambda_2^*, 1)$ equilibrium for some $\lambda_1^*, \lambda_2^* \in [1/2, 1/2]$ becomes stable when $\tau \in (0.381325, 0.382161)$. After that, a type $(\lambda_1^*, 1, 1)$ equilibrium for some $\lambda_1^* \in [1/2, 1]$ is stable when $\tau \in [0.377947, 0.381325]$, and the full agglomerating equilibrium $(1, 1, 1)$ is stable when $\tau \in [0.0181174, 0.377947]$. A type $(\lambda_1^*, 1, 1)$ equilibrium becomes stable again for some $\lambda_1^* \in (0, 1)$ when $\tau \in (0.00717398, 0.0181174)$. The completely separating equilibrium $(0, 1, 1)$ is stable if $\tau \in (0.00562582, 0.0077398)$, and, finally, type $(0, \lambda_2^*, 1)$ becomes stable for suitable $\lambda_2 \in (0, 1)$ when $\tau < 0.00562582$.

Figure 1 draws a typical evolution process for this example. Several remarks follow.

- Only the 3rd industry catastrophically changes from dispersion to agglomeration. Other industries move gradually. The bang-bang behavior of many spatial models disappears to some extent in the case of multi-manufacturing industries.

- Industry 1 first agglomerates in region n and then in region s. This describes the history of the U.S. tire industry, which was spectacularly localized in the city of Akron in Ohio (OH) before 1930 but no major producers of tires are now located in Akron. (Krugman [14], p.62-63).

- Different industries may disperse in different time periods. In this example, industry 1 only disperses when $\tau \in (0.00717398, 0.0181174)$ and industry 2 only disperses when $\tau < 0.00562582$. This is consistent with the fact that Silicon Valley rises while Detroit fades.

- We find that the dispersion order is determined by the labor intensities of industries in the re-dispersion process. In other words, the most labor-intensive industry disperses first, and the less labor-intensive ones disperse second. It is meaningful that the least labor-intensive industry does not disperse again once it agglomerates. The observation is consistent with Proposition 2. In real life, financial services make up one of the least labor-intensive industries, and they have actually agglomerated in New York, London, and Tokyo for a long time.

- If we view the manufacturing sector as a whole, the evolution process is just from dispersion to agglomeration and then to re-dispersion, as observed in the literature.

- The inner equilibrium of full dispersion appears only when $\tau > 0.382161$ in this example. If ϕ_1, ϕ_2 and ϕ_3 are close enough, this equilibrium may appear again for smaller τ, which is the same as the re-dispersion fact in the single industry model. However, this equilibrium will not last for all small τ if at least two ϕ_i and ϕ_j are different.

Of course, the process shown in the figure is not the only one. History and expectations play important roles in determining a particular process (Krugman [13]).

4 A general framework

Our principal results can be generalized to the case of asymmetric regions and positive agricultural transport cost τ_a. Let A^n and A^s be the numbers of unskilled workers in regions n and s, respectively. Without loss of generality, we let $A^n \geq A^s$. The prices of agricultural products are then different in two regions and become

$$\begin{aligned}
p_a^{nn} &= \frac{a_a}{b_a} + \frac{\sum_{i=1}^K (c_a + b_a\lambda_i)N\phi_i - c_a(A^n + A^s) - b_a A^n}{b_a(b_a + 2c_a)(A^n + A^s + L)} - \frac{A^s + \sum_{i=1}^K (1-\lambda_i)N}{A^n + A^s + L}\tau_a, \\
p_a^{ns} &= p_a^{nn} + \tau^a, \\
p_a^{ss} &= \frac{a_a}{b_a} + \frac{\sum_{i=1}^K [c_a + b_a(1-\lambda_i)]N\phi_i - c_a(A^n + A^s) - b_a A^s}{b_a(b_a + 2c_a)(A^n + A^s + L)} - \frac{A^n + \sum_{i=1}^K \lambda_i N}{A^n + A^s + L}\tau_a, \\
p_a^{sn} &= p_a^{ss} + \tau^a.
\end{aligned}$$

The consumption of those agricultural goods in two regions becomes

$$\begin{aligned}
q_a^{nn} =& \tau_a \left\{ \frac{(b_a + 2c_a)A^s + c_a L}{A^n + A^s + L} + \sum_{i=1}^K \frac{[-c_a + (b_a + 2c_a)(1-\lambda_i)]N}{A^n + A^s + L} \right\} \\
&+ \frac{A^n}{A^n + A^s + L} - \sum_{i=1}^K \frac{\lambda_i \phi_i N}{A^n + A^s + L},
\end{aligned}$$

$$
\begin{aligned}
q_a^{ns} =&\tau_a\Big\{-\frac{(b_a+2c_a)A^n+(b_a+c_a)L}{A^n+A^s+L}+\sum_{i=1}^K\frac{[-c_a+(b_a+2c_a)(1-\lambda_i)]N}{A^n+A^s+L}\Big\} \\
&+\frac{A^n}{A^n+A^s+L}-\sum_{i=1}^K\frac{\lambda_i\phi_iN}{A^n+A^s+L}, \\
q_a^{ss} =&\tau_a\Big\{\frac{(b_a+2c_a)A^n+c_aL}{A^n+A^s+L}+\sum_{i=1}^K\frac{[-c_a+(b_a+2c_a)\lambda_i]N}{A^n+A^s+L} \\
&+\frac{A^s}{A^n+A^s+L}-\sum_{i=1}^K\frac{(1-\lambda_i)\phi_iN}{A^n+A^s+L}, \\
q_a^{sn} =&\tau_a\Big\{-\frac{(b_a+2c_a)A^s+(b_a+c_a)L}{A^n+A^s+L}+\sum_{i=1}^K\frac{[-c_a+(b_a+2c_a)\lambda_i]N}{A^n+A^s+L}\Big\} \\
&+\frac{A^s}{A^n+A^s+L}-\sum_{i=1}^K\frac{(1-\lambda_i)\phi_iN}{A^n+A^s+L}.
\end{aligned}
$$

We suppose that all the prices and all the consumption amounts in two regions are positive. This is true if we impose

$$
\begin{aligned}
\tau_a < \min\Big\{&\frac{a_a(b_a+2c_a)(A^n+A^s+L)-c_aA^s-(b_a+c_a)A^n+c_aN\sum_{i=1}^K\phi_i}{b_a(b_a+2c_a)(A^s+L)}, \\
&\frac{a_a(b_a+2c_a)(A^n+A^s+L)-c_aA^n-(b_a+c_a)A^s+c_aN\sum_{i=1}^K\phi_i}{b_a(b_a+2c_a)(A^n+L)}, \\
&\frac{A^n-N\sum_{i=1}^K\phi_i}{(A^n+L)(b_a+2c_a)},\quad \frac{A^s-N\sum_{i=1}^K\phi_i}{(A^s+L)(b_a+2c_a)}\Big\}
\end{aligned}
\tag{27}
$$

Condition (27) degenerates to (6) if $\tau_a = 0$ and $A^n = A^s = A$.

Although the algebra is somewhat more difficult, we find that the utility difference (7) between skilled workers of type i in two regions has a similar expression:

$$
V_n^i(\boldsymbol{\lambda})-V_s^i(\boldsymbol{\lambda})=\sum_{j=1}^K(\frac{1}{2}-\lambda_j)\tilde{\delta}_{ij}-\xi_i,
$$

where

$$
\begin{aligned}
\tilde{\nu} =&\frac{(b_m+c_mN)}{2(2b_m+c_mN)^2}[6b_m^2+c_m^2N(A^n+A^s+L)+2b_mc_m(A^n+A^s+L+2N)]\tau^2 \\
&-\frac{2a_m(3b_m+2c_mN)(b_m+c_mN)}{(2b_m+c_mN)^2}\tau, \\
\tilde{\delta}_{ij} =&\begin{cases} N\Big\{\tilde{\nu}+\frac{2[\phi_i+(b_a+2c_a)\tau_a]^2}{(b_a+2c_a)(A^n+A^s+L)}\Big\} & \text{if } i=j, \\ N\Big\{\mu+\frac{2[\phi_i+(b_a+2c_a)\tau_a][\phi_j+(b_a+2c_a)\tau_a]}{(b_a+2c_a)(A^n+A^s+L)}\Big] & \text{if } i\neq j, \end{cases}
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
\xi_i =&(A^n-A^s)\Big\{\frac{(b_m+c_mN)}{2(2b_m+c_mN)}(b_m\tau-2a_m)\tau \\
&-\frac{[1-(b_a+2c_a)\tau_a][\phi_i+(b_a+2c_a)\tau_a]}{(b_a+2c_a)(A^n+A^s+L)}\Big\}.
\end{aligned}
\tag{29}
$$

Note that the second term in the curly braces of ξ_i is negative due to (27). Therefore, a larger ϕ_i or larger τ_a corresponds to a smaller ξ_i.

After comparing $\tilde{\delta}_{ij}$ with δ_{ij}, we know that a positive τ_a does not significantly change the model. What we need to do is only to replace ϕ_i by $\phi_i + [(b_a + 2c_a)\tau_a]$ for all industry i in the arguments.

However, a different population of unskilled workers makes the regions asymmetric. Solving $V_n^i(\boldsymbol{\lambda}) - V_s^i(\boldsymbol{\lambda}) = 0$ for all $i = 1, \ldots, K$ by Cramer's formulas, we obtain solution

$$\tilde{\lambda}_i^* = \frac{1}{2} - \frac{|\Xi, \tilde{\Delta}^{-i}|}{|\tilde{\Delta}|},$$

where $\Xi = (\xi_1, \ldots, \xi_K)'$, $\tilde{\Delta} = (\tilde{\delta}_{ij})_{K\times K}$, and (Ξ, Δ^{-i}) is the matrix where the column i of Δ is substituted by Ξ. $(\tilde{\lambda}_1^*, \ldots, \tilde{\lambda}_K^*)$ is the unique inner equilibrium if $\tilde{\lambda}_i^* \in (0,1)$ for all $i = 1, \ldots, K$. This equilibrium is stable if

$$\begin{aligned}\tilde{\Delta}_k =& N^k(\tilde{\nu} - \mu)^{k-2}\Big\{(\tilde{\nu} - \mu)[\tilde{\nu} + (k-1)\mu] + \frac{2(\tilde{\nu} - \mu)}{(b_a + 2c_a)(A^n + A^s + L)}\sum_{j=1}^{k}\phi_j^2 \\ &+ \frac{\mu}{(b_a + 2c_a)(A^n + A^s + L)}\sum_{i=1}^{k}\sum_{j=1}^{k}(\phi_i - \phi_j)^2\Big\} \\ >& 0\end{aligned} \tag{30}$$

holds for all $k = 1, \ldots, K$. Theorems 1 and 3 hold again. In particular, Theorem 3 leads to the following conclusion:

Proposition 6 *There is no stable equilibrium in which at least two industries disperse for sufficiently small τ.*

Proof: To the contrary, suppose that there is a stable equilibrium, in which $K_1 \geq 2$ industries disperse, K_2 industries locate in region n and $K_3 = K - K_1 - K_2$ industries locate in region s. Without loss of generality, let the equilibrium be $(\lambda_1^*, \ldots, \lambda_{K_1}^*, 1, \ldots, 1, 0, \ldots, 0)$, where $\lambda_k^* \in (0,1)$ for $k = 1, \ldots, K_1$. Then,

$$\sum_{j=1}^{K_1}(\frac{1}{2} - \lambda_j^*)\tilde{\delta}_{ij} - \tilde{\xi}_i = 0, \quad \imath = 1, \ldots, K_1,$$

where

$$\tilde{\xi}_i = \xi_i + \frac{1}{2}\sum_{j=K_1+1}^{K_1+K_2}\tilde{\delta}_{ij} - \frac{1}{2}\sum_{j=K_1+K_2+1}^{K}\tilde{\delta}_{ij},$$

and $\tilde{\delta}_{ij}$, ξ_i are defined in (28) and (29), respectively. Therefore, a necessary condition for the equilibrium to be stable is that $|\tilde{\Delta}_K|$ of (30) is positive, which is impossible when τ is small enough according to the asymmetric version of Theorem 3. □

On the other hand, Propositions 1-4 depend on the differential between A^n and A^s. For example, we may have full agglomeration if the regions are very different, when τ is small:

Proposition 7 *A full agglomeration equilibrium* $\mathbf{1} = (1, \ldots, 1)$ *is stable when* τ *is small if and only if*

$$A^n - A^s \geq \frac{N}{1-(b_a+2c_a)\tau_a} \sum_{j=1}^{K} [\phi_j + (b_a+2c_a)\tau_a]. \tag{31}$$

The proof is in Appendix I. This conclusion shows that an extremely small region with few unskilled workers may not escape from being a sparsely populated area forever.

5 Welfare analysis

We consider the second-best welfare, in which the planner is able to assign any number of skilled workers to a specific region but is not able to control the product prices. The planner maximizes the total welfare of all skilled workers and unskilled workers in two regions. Specifically, the welfare function is defined as

$$\begin{aligned}\text{Wel} =& \left(A^n + N\sum_{i=1}^{K}\lambda_i\right)(S_m^n + S_a^n) + A^n p_a^{nn} + N\sum_{i=1}^{K}\lambda_i w_i^n \\ &+ \left[A^s + N\sum_{i=1}^{K}(1-\lambda_i)\right](S_m^s + S_a^s) + A^s p_a^{ss} + N\sum_{i=1}^{K}(1-\lambda_i)w_i^s,\end{aligned}$$

where p_a^{nn} and p_a^{ss} are the prices of local agricultural goods in regions n and s, which are equal to the wages of unskilled workers in two regions, respectively. After a cumbersome simplification process, the welfare function can be rewritten as

$$\text{Wel} = N\left[-\frac{1}{2}\sum_{i=1}^{K}\sum_{j=1}^{K}\delta_{ij}^0\left(\frac{1}{2}-\lambda_i\right)\left(\frac{1}{2}-\lambda_j\right) + \sum_{i=1}^{K}\xi_i^0\left(\frac{1}{2}-\lambda_i\right)\right] + \text{constant}, \tag{32}$$

where

$$\begin{aligned}\nu^0 =& \frac{(b_m+c_mN)[24b_m^2 + 3c_m^2N(A^n+A^s+L) + 8b_mc_m(A^n+A^s+L+N)]}{4(2b_m+c_mN)^2}\tau^2 \\ &- \frac{4a_m(3b_m+c_mN)(b_m+c_mN)}{(2b_m+c_mN)^2}\tau, \\ \mu^0 =& \frac{2(b_m+c_mN)(3b_m+c_mN)}{(2b_m+c_mN)^2}(b_m\tau - 2a_m)\tau, \\ \delta_{ij}^0 =& \begin{cases} N\left\{\nu^0 + \frac{2[\phi_i+(b_a+2c_a)\tau_a]^2}{(b_a+2c_a)(A^n+A^s+L)}\right\} & \text{if } i = j \\ N\left\{\mu^0 + \frac{2[\phi_i+(b_a+2c_a)\tau_a][\phi_j+(b_a+2c_a)\tau_a]}{(b_a+2c_a)(A^n+A^s+L)}\right\} & \text{if } i \neq j, \end{cases} \\ \xi_i^0 =& (A^n-A^s)\left\{\frac{(3b_m+c_mN)(b_m+c_mN)}{2(2b_m+c_mN)^2}(b_m\tau-2a_m)\tau \right. \\ &\left. - \frac{[1-(b_a+2c_a)\tau_a][\phi_i+(b_a+2c_a)\tau_a]}{(b_a+2c_a)(A^n+A^s+L)}\right\}.\end{aligned}$$

(The detail of the computation is available from the author.)

Since

$$\nu^0 - \mu^0 = \frac{c_m(A^n + A^s + L)(8b_m + 3c_m N)(b_m + c_m N)\tau^2}{4(2b_m + c_m N)^2} > 0,$$

we find that (32) is similar to (11). Furthermore, the expressions of δ_{ij}^0 and ξ_i^0 are similar to the expressions of $\tilde{\delta}_{ij}$ and ξ of Section 4. Therefore, the conclusions for the evolution of regions obtained in equilibrium analysis hold again for optimality. In particular, the difference between dispersion and re-dispersion remains in the viewpoint of social optimality. However, due to the discrepancy between expressions (32) and (11), we know that the equilibrium and optimum do not completely coincide. In other words, even though skilled workers have individual incentives to move, these incentives do no reflect the view of the social value.

6 Conclusion

The core-periphery model of Krugman [12], which is the base of New Economic Geography, is illustrative and simple. This pioneering work is extended in this paper by considering multiple industries. A contribution of this paper is to provide a general equilibrium model while maintaining high tractability. While the core-periphery model clarifies how the whole manufacturing sector evolves when the transport cost decreases, this paper reveals how each individual manufacturing industry evolves. Surprisingly, we find that the interaction of demand, increasing returns, and transportation costs is sufficient to derive a separating equilibrium, in which different industries agglomerate in different regions. We believe that the results here could partly explain why Detroit emerged as the automotive center, Seattle as the aircraft center, Rochester as the photographic equipment center, New York as the garment center, and Grand Rapids as the furniture center.

Fujita et al. [7] is the first paper considering the case of multiple industries. They show that a Christaller-type hierarchical urban system emerges in a self-organizing manner when the population size of the economy gradually increases. A complementary result is provided in this paper, clarifying how regions evolve when the transport cost decreases gradually. It is interesting that both of them suggest that economies of agglomeration must be considered at the industry level.

Since the situation of multiple industries is very complicated, we do not affirm that this model captures all of the features. For example, we model different industries by the numbers of unskilled workers in their productions. The transport costs for different industries are supposed to be the same; however, they are different in the real world. This suggests an interesting future extension of our approach. In addition, one region may contain more than one agglomerated industries in our model. This is due to the assumption of two regions. Recently, Tabuchi, Thisse and Zeng [22] extend the core-periphery model to a multi-regional framework and clarify how large, middle-sized and small regions evolve when the transport costs decrease. More will definitely be revealed when we study a combination of these two models and analyze the situation of multiple industries in multiple regions in future research.

Nevertheless, it is shown in this paper that the framework of Ottaviano et al. [17] is powerful enough to be generalized to analyze surprisingly complex systems.

Appendix A: The proof of (10)

Let

$$\Upsilon_k = \begin{bmatrix} \nu & \mu & \cdots & \mu \\ \mu & \nu & \cdots & \mu \\ \vdots & \vdots & \ddots & \vdots \\ \mu & \mu & \cdots & \nu \end{bmatrix}_{k\times k}, \quad \Phi_k = [\phi_1, \dots, \phi_k]'$$

and $(\Phi_k, \Upsilon_k^{-i})$ be the matrix of Υ_k where the ith column is replaced by Φ_k.

Then, for $k = 1, \dots, K$,

$$\begin{aligned} |\Delta_k| = & \begin{vmatrix} \delta_{11} & \cdots & \delta_{1k} \\ \vdots & \ddots & \vdots \\ \delta_{k1} & \cdots & \delta_{kk} \end{vmatrix} \\ =& N^k|\Upsilon_k| + \frac{2N^k}{(b_a+2c_a)(2A+L)}\sum_{i=1}^k \phi_i|\Phi_k, \Upsilon_k^{-i}| \\ =& N^k[(\nu+(k-1)\mu](\nu-\mu)^{k-1} \\ &+ \frac{(\nu-\mu)^{k-2}N^k}{(b_a+2c_a)(2A+L)}\left[2(\nu-\mu)\sum_{i=1}^k \phi_i^2 + \sum_{i,j=1}^k(\phi_i-\phi_j)^2\mu\right] \\ =& N^k(\nu-\mu)^{k-2}\Bigg\{(\nu-\mu)[\nu+(k-1)\mu] + \frac{2(\nu-\mu)}{(b_a+2c_a)(2A+L)}\sum_{i=1}^k\phi_i^2 \\ &+ \frac{\mu}{(b_a+2c_a)(2A+L)}\sum_{i=1}^k\sum_{j=1}^k(\phi_i-\phi_j)^2\Bigg\}. \end{aligned}$$

Appendix B: The proof of Theorem 1

According to (10), we have $|\Delta_K| \neq 0$ except for finite number of τ. So it generically holds that $\Delta_K \neq 0$. Then there is a full rank matrix T such that

$$\mathcal{P}(\boldsymbol{\lambda}) = (T(\frac{\mathbf{1}}{\mathbf{2}} - \boldsymbol{\lambda}))' \begin{pmatrix} \varepsilon_1 & 0 & \cdots & 0 \\ 0 & \varepsilon_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \varepsilon_K \end{pmatrix} (T(\frac{\mathbf{1}}{\mathbf{2}} - \boldsymbol{\lambda})), \tag{33}$$

where $\varepsilon_i \neq 0$ for all $i = 1, \dots, K$. On the other hand, we have

$$T\bigg(\frac{\mathbf{1}}{\mathbf{2}} - \boldsymbol{\lambda}\bigg) \in T\bigg(\bigg[-\frac{1}{2}, \frac{1}{2}\bigg]^K\bigg) \equiv \bigg\{T\mathbf{x} \mid \mathbf{x} \in \bigg[-\frac{1}{2}, \frac{1}{2}\bigg]^K\bigg\},$$

because

$$\frac{\mathbf{1}}{\mathbf{2}} - \boldsymbol{\lambda} \in \bigg[-\frac{1}{2}, \frac{1}{2}\bigg]^K.$$

Since each $\varepsilon_i \neq 0$ for all i, it is easy to see that $\varepsilon_1 y_1^2 + \ldots + \varepsilon_K y_K^2$ has a locally unique maximizer $\mathbf{y}^* = (y_1^*, \dots, y_K^*) \in T([-\frac{1}{2}, \frac{1}{2}]^K)$ by induction on K, so $\mathcal{P}(\boldsymbol{\lambda})$ has a locally unique maximizer $\boldsymbol{\lambda}^* = \frac{\mathbf{1}}{\mathbf{2}} - T^{-1}(\mathbf{y}^*)$ from (33). □

Appendix C: Proof of Theorem 2

Contrary to the conclusion, we suppose that there is a stable separating equilibrium. Without loss of generality, assume that industries $1, \ldots, K_1$ agglomerate in region n, industries $K_1 + 1, \ldots, K_1 + K_2$ disperse between two regions, industries $K_1 + K_2 + 1, \ldots, K_1 + K_2 + K_3 = K$ agglomerate in region s, and $K_1 \geq K_3 \geq 1$. There are two cases. First, $K_2 = 0$ holds if no industry disperses. Denote the equilibrium as $\boldsymbol{\lambda}^{s*} = (1, \ldots, 1, 0, \ldots, 0)$. By calculation, we have

$$\begin{aligned}
V_1^n(\boldsymbol{\lambda}^{s*}) - V_1^s(\boldsymbol{\lambda}^{s*}) &= -\frac{1}{2}\sum_{j=1}^{K_1} \delta_{1j} + \frac{1}{2}\sum_{j=K_1+1}^{K_1+K_3} \delta_{1j} \\
&= -\frac{\nu - \mu}{2} + \frac{2(K_3 - K_1)\phi^2}{(b_a + 2c_a)(2A + L)} \leq -\frac{\nu - \mu}{2} < 0,
\end{aligned}$$

where the first inequality comes from $K_3 \geq K_1$ and $\phi \geq 0$, the second inequality comes from (4). Therefore equilibrium $\boldsymbol{\lambda}^{s*}$ is unstable. Second, if there is at least one dispersing industry, then $K_2 \geq 1$. Let $\lambda_i^* \in (0, 1)$ be the proportion of type $i = K_1 + 1, \ldots, K_1 + K_2$ skilled workers in region n in the equilibrium and the equilibrium is $\boldsymbol{\lambda}^{d*} = (1, \ldots, 1, \lambda^*_{K_1+1}, \ldots, \lambda^*_{K_1+K_2}, 0, \ldots, 0)$. Then

$$\begin{aligned}
V_1^n(\boldsymbol{\lambda}^{d*}) - V_1^s(\boldsymbol{\lambda}^{d*}) =& -\frac{1}{2}\sum_{j=1}^{K_1} \delta_{1j} + \sum_{j=K_1+1}^{K_1+K_2}\left(\frac{1}{2} - \lambda_j^*\right)\delta_{1j} + \frac{1}{2}\sum_{j=K_1+K_2+1}^{K} \delta_{1j} \\
=& -\frac{1}{2}(\nu - \mu) + \sum_{j=1}^{K_1} \delta_{K_1+1,j} + \left(\frac{1}{2} - \lambda^*_{K_1+1}\right)(\mu - \nu) \\
&+ \sum_{j=K_1+1}^{K_1+K_2}\left(\frac{1}{2} - \lambda_j^*\right)\delta_{K_1+1,j} + \frac{1}{2}\sum_{j=K_1+K_2+1}^{K} \delta_{K_1+1,j} \\
=& V^n_{K_1+1}(\boldsymbol{\lambda}^{d*}) - V^s_{K_1+1}(\boldsymbol{\lambda}^{d*}) - (1 - \lambda^*_{K_1+1})(\nu - \mu) \\
=& -(1 - \lambda^*_{K_1+1})(\nu - \mu) < 0,
\end{aligned}$$

where the inequality is from $\lambda^*_{K_1+1} < 1$ and (4). Therefore, equilibrium $\boldsymbol{\lambda}^{d*}$ is also unstable. □

Appendix D: Proof of Proposition 1

For the notation convenience, we assume that $\tau_{\text{trade}} > \tau_\sharp^2$ when $\tau_\sharp^2$ exists. If $\phi > \phi_\sharp$ or $\phi < \phi_\sharp$ but $\tau \in (\tau_\sharp^1, \tau_\sharp^2)$, then

$$\nu + (K - 1)\mu + \frac{2K\phi^2}{(b_a + 2c_a)(2A + L)} < 0,$$

and therefore, (14) is positive for all i, which shows that full agglomeration is stable. Finally, when $\phi < \phi_\sharp$ but $\tau \in (0, \tau_\sharp^1) \cup (\tau_\sharp^1, \tau_\sharp^2)$, it holds that

$$\nu + (K - 1)\mu + \frac{2K\phi^2}{(b_a + 2c_a)(2A + L)} > 0. \tag{34}$$

There are two cases. First, if

$$\mu + \frac{2\phi^2}{(b_a + 2c_a)(2A + L)} \geq 0, \tag{35}$$

then

$$\nu - \mu + k\left[\mu + \frac{2\phi^2}{(b_a + 2c_a)(2A + L)}\right] > 0$$

holds because $\nu - \mu > 0$. Therefore, (13) is positive for all k. Second, if (35) does not hold, then

$$\nu - \mu + k\left[\mu + \frac{2\phi^2}{(b_a + 2c_a)(2A + L)}\right] \geq \nu - \mu + K\left[\mu + \frac{2\phi^2}{(b_a + 2c_a)(2A + L)}\right] > 0,$$

where the last inequality holds from (34). Therefore (13) is positive again. The above shows that, in either case, the full dispersion is stable. □

Appendix E: Proof of Proposition 2

Contrary to the second inequality of (16), we suppose $\lambda^*_{i_2} > \max\{\lambda^*_{i_1}, \lambda^*_{i_3}\}$. Then $\lambda^*_{i_2} > 0$, $\lambda^*_{i_1} < 1$ and $\lambda^*_{i_3} < 1$. Therefore, from (7), we have

$$\sum_{i \neq i_1, i_2, i_3} \left(\frac{1}{2} - \lambda^*_i\right)\delta_{i_1,i} + \left(\frac{1}{2} - \lambda^*_{i_1}\right)\delta_{i_1,i_1} + \left(\frac{1}{2} - \lambda^*_{i_2}\right)\delta_{i_1,i_2} + \left(\frac{1}{2} - \lambda^*_{i_3}\right)\delta_{i_1,i_3} \leq 0, \tag{36}$$

$$\sum_{i \neq i_1, i_2, i_3} \left(\frac{1}{2} - \lambda^*_i\right)\delta_{i_2,i} + \left(\frac{1}{2} - \lambda^*_{i_1}\right)\delta_{i_2,i_1} + \left(\frac{1}{2} - \lambda^*_{i_2}\right)\delta_{i_2,i_2} + \left(\frac{1}{2} - \lambda^*_{i_3}\right)\delta_{i_2,i_3} \geq 0, \tag{37}$$

$$\sum_{i \neq i_1, i_2, i_3} \left(\frac{1}{2} - \lambda^*_i\right)\delta_{i_3,i} + \left(\frac{1}{2} - \lambda^*_{i_1}\right)\delta_{i_3,i_1} + \left(\frac{1}{2} - \lambda^*_{i_2}\right)\delta_{i_3,i_2} + \left(\frac{1}{2} - \lambda^*_{i_3}\right)\delta_{i_3,i_3} \leq 0. \tag{38}$$

Subtracting (36) and (38) from (37), respectively, we obtain

$$(\lambda^*_{i_1} - \lambda^*_{i_2})(\nu - \mu) + \frac{(\phi_{i_2} - \phi_{i_1})[(1 - 2\lambda^*_{i_1})\phi_{i_1} + (1 - 2\lambda^*_{i_2})\phi_{i_2} + (1 - 2\lambda^*_{i_3})\phi_{i_3}]}{(b_a + 2c_a)(2A + L)} \geq 0, \tag{39}$$

$$(\lambda^*_{i_3} - \lambda^*_{i_2})(\nu - \mu) + \frac{(\phi_{i_2} - \phi_{i_3})[(1 - 2\lambda^*_{i_1})\phi_{i_1} + (1 - 2\lambda^*_{i_2})\phi_{i_2} + (1 - 2\lambda^*_{i_3})\phi_{i_3}]}{(b_a + 2c_a)(2A + L)} \geq 0. \tag{40}$$

It holds from (39) that

$$\frac{(\phi_{i_2} - \phi_{i_1})[(1 - 2\lambda^*_{i_1})\phi_{i_1} + (1 - 2\lambda^*_{i_2})\phi_{i_2} + (1 - 2\lambda^*_{i_3})\phi_{i_3}]}{(b_a + 2c_a)(2A + L)} \geq (\lambda^*_{i_2} - \lambda^*_{i_1})(\nu - \mu) > 0$$

where the last inequality is from (4) and the assumption that $\lambda^*_{i_2} > \max\{\lambda^*_{i_1}, \lambda^*_{i_3}\}$. Furthermore, from (15), we have

$$(1 - 2\lambda^*_{i_1})\phi_{i_1} + (1 - 2\lambda^*_{i_2})\phi_{i_2} + (1 - 2\lambda^*_{i_3})\phi_{i_3} < 0. \tag{41}$$

Similarly, (40) derives

$$(1 - 2\lambda^*_{i_1})\phi_{i_1} + (1 - 2\lambda^*_{i_2})\phi_{i_2} + (1 - 2\lambda^*_{i_3})\phi_{i_3} > 0,$$

which contradicts (41). Therefore, the second inequality of (16) is true. The first inequality of (16) can be shown in a similar way.

Appendix F: Proof of Theorem 3

We rewrite Δ_K as follows.

$$\Delta_K = N^K(\nu-\mu)^{K-1}\Big[\nu+(K-1)\mu+\frac{2\sum_{i=1}^K\phi_i^2}{(b_a+2c_a)(2A+L)} + \frac{\mu\sum_{i=1}^K\sum_{j=1}^K(\phi_i-\phi_j)^2}{(\nu-\mu)(b_a+2c_a)(2A+L)}\Big]. \tag{42}$$

The last term is nonzero because $K \geq 2$ and at least one inequality in (15) holds strictly. Furthermore, the following term

$$\frac{\mu}{\nu-\mu} = \frac{2(2b_m+c_mN)(3b_m+2c_mN)}{c_m(A_1+A_2+L)}\frac{b_m\tau-2a_m}{\tau}$$

converges to $-\infty$ when $\tau \to 0$. Therefore, (42) is negative when τ is small enough, and hence there is no inner stable equilibrium of (8).

Appendix G: Proof of Proposition 3

Let $\lambda^*_{i^d}$ be the solution of

$$-\frac{1}{2}\delta_{i^d,1}-\ldots-\frac{1}{2}\delta_{i^d,i^d-1}+(\frac{1}{2}-\lambda^*_{i^d})\delta_{i^d,i^d}+\frac{1}{2}\delta_{i^d,i^d+1}+\ldots+\frac{1}{2}\delta_{i^d,K}=0. \tag{43}$$

From (17) and (18), we know immediately that $\lambda^*_{i^d} \in [0,1)$ holds for sufficiently small τ. Since $\lim_{\tau\to 0}\nu = 0$ and $\phi_{i^d} > 0$, we have $\lim_{\tau\to 0}\delta_{i^d,i^d} > 0$, so equilibrium $(1,\ldots,1,\lambda^*_{i^d},0,\ldots,0)$ is stable if

$$-\frac{1}{2}\delta_{j1}-\ldots-\frac{1}{2}\delta_{j,i^d-1}+(\frac{1}{2}-\lambda^*_{i^d})\delta_{j,i^d}+\frac{1}{2}\delta_{j,i^d+1}+\ldots+\frac{1}{2}\delta_{j,K}>0 \text{ for } j=1,\ldots,i^d-1, \tag{44}$$

and

$$-\frac{1}{2}\delta_{j1}-\ldots-\frac{1}{2}\delta_{j,i^d-1}+(\frac{1}{2}-\lambda^*_{i^d})\delta_{j,i^d}+\frac{1}{2}\delta_{j,i^d+1}+\ldots+\frac{1}{2}\delta_{j,K}<0 \text{ for } j=i^d+1,\ldots,K. \tag{45}$$

By solving $\lambda^*_{i^d}$ from (43), (44) is equivalent to

$$\begin{vmatrix} \delta_{j1}+\ldots+\delta_{j,i^d-1}-\delta_{j,i^d+1}-\ldots-\delta_{j,K} & \delta_{j,i^d} \\ \delta_{i^d,1}+\ldots+\delta_{i^d,i^d-1}-\delta_{i^d,i^d+1}-\ldots-\delta_{i^d,K} & \delta_{i^d,i^d} \end{vmatrix} < 0. \tag{46}$$

The LHS of (46) is

$$\mu\Big\{\Big(\frac{\nu}{\mu}-1\Big)\Big[\nu+(2i^d-K-1)\mu+\frac{2\phi_j(\phi_1+\ldots+\phi_{i^d}-\phi_{i^d+1}-\ldots-\phi_K)}{(b_a+2c_a)(2A+L)}\Big] + \frac{2(\phi_j-\phi_{i^d})}{(b_a+2c_a)(2A+L)}\Big[1-\frac{\nu}{\mu}+\sum_{i=1}^{i^d-1}(\phi_i-\phi_{i^d})+\sum_{i=i^d+1}^{K}(\phi_{i^d}-\phi_i)\Big]\Big\}. \tag{47}$$

Since $\lim_{\tau \to 0} \nu/\mu = 1$, the first item of (47) converges to 0 when τ is small. In contrast, the second item of (47) is definitely positive for $j < i^d$ because of (19), and therefore, inequality (46) holds for sufficient small τ.

On the other hand, (45) is equivalent to

$$\begin{vmatrix} \delta_{j1} + \ldots + \delta_{j,i^d-1} - \delta_{j,i^d+1} - \ldots - \delta_{j,K} & \delta_{j,i^d} \\ \delta_{i^d,1} + \ldots + \delta_{i^d,i^d-1} - \delta_{i^d,i^d+1} - \ldots - \delta_{i^d,K} & \delta_{i^d,i^d} \end{vmatrix} > 0.$$

The LHS of the above expression is also (47), which is actually positive for $j > i^d + 1$ such that $\phi_j < \phi_{i^d}$ and small τ. For those j such that $\phi_j = \phi_{i^d} (> 0)$, (47) becomes

$$(\nu - \mu)\left[\nu + (2i^d - K - 1)\mu + \frac{2\phi_j(\phi_1 + \ldots + \phi_{i^d} - \phi_{i^d+1} - \ldots - \phi_K)}{(b_a + 2c_a)(2A + L)}\right].$$

The term inside square brackets is positive for sufficient small τ, and the above expression is therefore positive again.

Appendix H: Proof of Proposition 5

(i). Equilibria $(0, 1, 1)$ and $(1, 0, 0)$ are stable if and only if

$$\delta_{11} - \delta_{12} - \delta_{13} < 0, \tag{48}$$
$$\delta_{21} - \delta_{22} - \delta_{23} > 0, \tag{49}$$
$$\delta_{31} - \delta_{32} - \delta_{33} > 0. \tag{50}$$

It is easy to check that (48) and (49) can be rewritten as (21). Furthermore, the first part of (21) implies that $\phi_1 < \phi_1 + \phi_2$, together with which (49) implies (50).

(ii). Necessity Without loss of generality, we assume a stable equilibrium $(\lambda_1^*, 1, 1)$ where industries 2 and 3 are all located in region n. Since $(\lambda_1^*, 1, 1)$ is an equilibrium distribution, λ_1^* should solve

$$(\frac{1}{2} - \lambda_1^*)\delta_{11} - \frac{1}{2}\delta_{12} - \frac{1}{2}\delta_{13} = 0,$$

and furthermore $\lambda_1^* \in (0, 1)$. Therefore,

$$\lambda_1^* = \frac{\delta_{11} - \delta_{12} - \delta_{13}}{2\delta_{11}}. \tag{51}$$

One of the necessary conditions for this equilibrium to be stable is that $\delta_{11} > 0$. So $\lambda_1^* \in (0, 1)$ can be rewritten as (22). Furthermore, since equilibrium $(\lambda_1^*, 1, 1)$ is stable, we also have

$$(\frac{1}{2} - \lambda_1^*)\delta_{21} - \frac{1}{2}\delta_{22} - \frac{1}{2}\delta_{23} > 0, \tag{52}$$
$$(\frac{1}{2} - \lambda_1^*)\delta_{31} - \frac{1}{2}\delta_{32} - \frac{1}{2}\delta_{33} > 0. \tag{53}$$

It is easy to check that (52) can be rewritten as (23).

Sufficiency Let λ_1^* be defined by (51), we prove that $(\lambda_1^*, 1, 1)$ is a stable equilibrium if (22) and (23) are true. Since $\delta_{11} > \delta_{12} \geq \delta_{13}$, the first inequality of (22) implies $\delta_{11} > 0$. Therefore

(22) implies that $\lambda_1^* \in (0,1)$, so $(\lambda_1^*, 1, 1)$ is an equilibrium. To show that it is also stable, we need to check (52) and (53). As mentioned before, (52) is equivalent to (23). Furthermore, since

$$\Delta_2 - \begin{vmatrix} \delta_{11} & \delta_{13} \\ \delta_{31} & \delta_{33} \end{vmatrix} = \frac{2N^2(\phi_2 - \phi_3)[(\nu - \mu)(\phi_2 + \phi_3) - \mu(\phi_1 - \phi_2) - \mu(\phi_1 - \phi_3)]}{(b_a + 2c_a)(2A + L)} \geq 0,$$

we know that (53) is implied by (52).

(iii) Necessity Without loss of generality, we assume a stable equilibrium $(\lambda_1^*, \lambda_2^*, 1)$, where industry 3 is located in region n. The following two equations determine λ_1^* and λ_2^*:

$$\begin{cases} V_1^n(\lambda_1^*, \lambda_2^*, 1) - V_1^s(\lambda_1^*, \lambda_2^*, 1) = (1/2 - \lambda_1^*)\delta_{11} + (1/2 - \lambda_2^*)\delta_{12} - (1/2)\delta_{13} = 0, \\ V_2^n(\lambda_1^*, \lambda_2^*, 1) - V_2^s(\lambda_1^*, \lambda_2^*, 1) = (1/2 - \lambda_1^*)\delta_{21} + (1/2 - \lambda_2^*)\delta_{22} - (1/2)\delta_{23} = 0, \end{cases}$$

which result in

$$\lambda_1^* = \frac{\Delta_2 - \begin{vmatrix} \delta_{13} & \delta_{12} \\ \delta_{23} & \delta_{22} \end{vmatrix}}{2\Delta_2}, \quad \lambda_2^* = \frac{\Delta_2 - \begin{vmatrix} \delta_{11} & \delta_{13} \\ \delta_{12} & \delta_{23} \end{vmatrix}}{2\Delta_2}. \tag{54}$$

Since $\lambda_1^*, \lambda_2^* \in (0,1)$, we have (26) and

$$-\Delta_2 < \begin{vmatrix} \delta_{11} & \delta_{12} \\ \delta_{31} & \delta_{32} \end{vmatrix} < \Delta_2. \tag{55}$$

The first inequality of (55) implies (25). Furthermore, due to the stability of this equilibrium, we have $\delta_{11} > 0$ and $\Delta_2 > 0$. Furthermore,

$$\begin{aligned} 0 <& V_3^n(\lambda_1^*, \lambda_2^*, 1) - V_3^s(\lambda_1^*, \lambda_2^*, 1) = (1/2 - \lambda_1^*)\delta_{31} + (1/2 - \lambda_2^*)\delta_{32} - (1/2)\delta_{33} \\ =& \frac{1}{2\Delta_2}\left(\delta_{13}\begin{vmatrix} \delta_{13} & \delta_{12} \\ \delta_{23} & \delta_{22} \end{vmatrix} + \delta_{23}\begin{vmatrix} \delta_{11} & \delta_{12} \\ \delta_{13} & \delta_{23} \end{vmatrix} - \delta_{33}\begin{vmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{vmatrix}\right) \\ =& -\frac{\Delta_3}{2\Delta_2}, \end{aligned}$$

which derives (24).

Sufficiency. If (24)-(26) hold, we define λ_1^* and λ_2^* by (54). Note that

$$\begin{vmatrix} \delta_{11} & \delta_{13} \\ \delta_{12} & \delta_{23} \end{vmatrix} + \begin{vmatrix} \delta_{12} & \delta_{13} \\ \delta_{22} & \delta_{23} \end{vmatrix} = \frac{2N^2(\phi_1 - \phi_2)[\mu(\phi_1 + \phi_2 - 2\phi_3) - (\nu - \mu)\phi_3]}{(b_a + 2c_a)(2A + L)} < 0,$$

we know that the second inequality of (54) is implied by the first inequality of (26). Therefore, $\lambda_1^*, \lambda_2^* \in (0,1)$ because of (25), (26), and $\Delta_2 > 0$. Furthermore, (24) implies that $V_3^n(\lambda_1^*, \lambda_2^*, 1) - V_3^s(\lambda_1^*, \lambda_2^*, 1) > 0$ so that this equilibrium is stable.

Appendix I: Proof of Proposition 7

Full agglomeration equilibrium **1** is stable if and only if

$$V_i^n(\mathbf{1}) - V_i^s(\mathbf{1}) = -\frac{N}{2}\left\{\tilde{\nu} + (K-1)\tilde{\mu} + \frac{2[\phi_i + (b_a + 2c_a)\tau_a]\sum_{j=1}^K[\phi_j + (b_a + 2c_a)\tau_a]}{(b_a + 2c_a)(A^n + A^s + L)}\right\}$$

$$
+ (A^n - A^s)\left\{\frac{[1-(b_a+2c_a)\tau_a][\phi_i+(b_a+2c_a)\tau_a]}{(b_a+c_a)(A^n+A^s+L)} - \frac{(b_m+c_mN)}{2(2b_m+c_mN}(b_m\tau-2a_m)\tau\right\}
$$
$$
>0, \quad i=1,\dots,K. \tag{56}
$$

When $\tau \to 0$, the LHS of (56) converges to

$$
\frac{\phi_i+(b_a+2c_a)\tau_a}{(b_a+2c_a)(A^n+A^s+L)}\left\{(A^n-A^s)[1-(b_a+2c_a)\tau_a] - N\sum_{j=1}^{K}[\phi_j+(b_a+2c_a)\tau_a]\right\},
$$

which is actually positive if (31) holds strictly and $\phi_i + (b_a + 2c_a)\tau_a > 0$. On the other hand, if either (31) holds equally or $\phi_i + (b_a + 2c_a)\tau_a = 0$, then the LHS of (56) is

$$
\frac{N}{2}[(-\tilde{\nu}) + (K-1)(-\tilde{\mu})] + (A^n - A^s)\frac{(b_m+c_mN)}{2(2b_m+c_mN)}(2a_m - b_m\tau)\tau,
$$

which is positive again when τ is small. Finally, if (31) does not hold, then the LHS of (56) converges to a negative number, and hence full agglomeration equilibrium is unstable when τ is small.

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