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# Risk-Averse Agents with Inequity Aversion, Ex Ante Contracting and Adverse Selection*

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## Abstract

In this paper, we introduce that the principal and the agent can contract at the *ex ante* stage, and allow for risk-averse agents with inequity aversion to analyze the properties of the optimal incentive scheme under adverse selection. Under inequity restrictive conditions, ex ante contracting structures often differ from those predicted by standard solutions of canonical adverse selection problem and the case of contracts offered at interim stage with inequity aversion.

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KEYWORDS: *Ex ante contracting, Inequity aversion, Risk-averse agent, Information rent. Adverse selection*

*This is very preliminary and incomplete. Comments are welcome.

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# 1 Introduction

Recent research into social preferences suggests that individual behavior is not only guided by payoff-equalizing motivations but also by concerns for overall welfare. In particular, Fehr and Schmidt (1999) find that the impact of fairness and inequity aversion depends on what workers compare. The theoretical works introducing inequity aversion into moral hazard problem were conducted by Bartling and Siemens (2004), Biel (2003), Englmaier and Wambach (2002), Grund and Sliwka (2002), Itoh (2004), Neilson and Stowe (2004), among others, and into adverse selection problem were conducted by Sappington (2004), Siemens (2004). While these papers introducing inequity aversion show important effects of horizontal (agents compare their welfare among themselves) or vertical comparison (agents compare their welfare to their principal's), in the most of inequity dealing with adverse selection, they consider the case of contracting offered at interim stage, i.e., once the agent already knows his type.

In the existing literature which analyzes the problems of inequity aversion with moral hazard and adverse selection under asymmetric information, it has either been assumed that risk-neutral with inequity averse agents or the case of contracting offered at interim stage have been analyzed. Instead in this paper, we introduce that the principal and the agent can contract at the *ex ante* stage, i.e., before the agent discovers his type, and allow for risk-averse agents with inequity aversion to analyze the properties of the optimal incentive scheme under adverse selection. To provide the simple closed-form expression of the optimum, we follow the inequity aversion of Siemens (2004)'s adverse selection rather than the framework of Sappington (2004), to a situation in which risk is introduced.

Due to introducing concerns with inequity into adverse selection, optimal incentive contract structures often differ from those predicted by standard solutions of canonical adverse selection problem. Contrary to the solutions of standard adverse selection problems, our first finding is that the efficient agent who is a risk-averse in presence of concerns with inequity produces more than first-best level, whereas the inefficient agent produces less than second-best level than pure adverse selection output. Creating a wedge between reward and punishment makes agent bear some risk. To guarantee the participation of the risk-averse agent with inequity aversion, the principal must pay not only a risk premium but also a wage premium for inequity aversion. Reducing these premiums call for both downward and upward distortion in both agents. Hence risk-aversion in presence of concern with inequity leads the principal to weaken incentive for inefficient agents, and to strengthen for efficient agents if they compare wages. Furthermore, increasing inequity induces the efficient agent's output to be high and makes the inefficient agent's output be small. Hence, there exists cutoff level which makes the cost functions be identical. As both types of agents receive the same wage there is no inequity where ex post informations are identical. When the efficient agent produces more than the first-best level of output, the inefficient agent's envy becomes large and his output is smaller and the efficient agent is larger. This makes the cutoff level be smaller when agent's envy becomes large.

Second, when risk-averse agents compare their information rents among themselves, optimal outputs in this model are the same as with traditional adverse selection framework of ex ante participation constraint. But ex post information rents differ from those analyzed by standard solutions of adverse selection and Siemens (2004). The first-best output of efficient agent takes the same form as is usual in adverse selection problems. From the result of first finding, when agents compare rents, the overall utility of the efficient agent is always larger than that of the inefficient, which the inefficient agent never get rent and asymmetric information and screening cause a rent inequity. Therefore, it becomes the incentive compatibility constraint as for which an efficient agent does not cause feel inequity. As a result, as the degree which inefficient agents envy is larger under some condition, the punishment becomes larger by the incentive compatibility constraint of the inefficient agent containing inequity.

Third, for the case when there is no inequity of standard adverse selection, we get the result of comparison as follows. Due to introducing inequity aversion into standard adverse selcetion, the optimal difference of ex post information rent is always smaller than he optimal difference of ex post information rent in canonical adverse selection if agents compare wages, whereas its direction reserves if agents compare rents. Since it is that an efficient agent also has envy to an inefficient agent's wages when comparing only wages, the difference of ex post information rent is smaller than the standard solutions of adverse selection. However, the efficient agents get a weakly higher rent so that only inefficient agents suffer from inequity if agents compare rents. it induces the difference of ex post information rent to be large than the standard solutions of adverse selection.

The paper is organized as follows. Section 2 presents the model of adverse selection in presence of concern with inequity of two types of risk-averse agent. In Section 3 analyzes the optimal incentive contract. Section 4 discusses applications and a brief conclusion.

## 2 The Model

Consider a risk-neutral principal facing a continuum of agents with measure one and the principal wants to delegate to an agent the production of $x$ units of a good. The value for the principal of these $x$ units is $S(x)$ where $S^{'}(x) > 0, S^{''}(x) < 0$ and $S(0) = 0$. The value of these goods to the principal is represented by a strictly increasing, strictly concave function $S(x)$. To ensure that some production will take place in every type, we assume that $S^{'}(0) = \infty$, and to ensure that production is always finite, we assume that $\lim_{x \to +\infty} S^{'}(x) = 0$.

The production cost of the agent is unobservable to the principal, but it is common knowledge that there two types of agents, $T = \{t_0, t_1\}$ with $0 < t_0 < t_1$. The agents of type $t_0$ are called efficient whereas agents of type $t_1$ are called inefficient. In other words, each agent can produce goods $x$ with type dependent cost function $C(x, t_0) = t_0 x$ or $C(x, t_1) = t_1 x$. The economic variables of an employed agent's relationship with the principal we consider are the quantity produced $x$ and the wage $w$ received by the agent. We assume that a risk-averse agent with a Von Neumann-Morgenstern utility function $u(\cdot)$ defined on his monetary gains $w - tx$,:

$$u(w - tx)$$

such that $u^{'} > 0, u^{''} < 0$ and $u(0) = 0$.

In our dealing with the case of adverse selection, we suppose that the case of contracts offered at the ex ante stage, i.e., before the agent discovers his type. The timing in the model is as follows.

- At the beginning of period 1 each worker's type is assumed to be unknown and the principal proposes a menu of contract $\{(w_0, x_0), (w_1, x_1)\}$: If $t_0$ (resp. $t_1$), the principal offers the wage $w_0$(resp. $w_1$) for the production $x_0$ (resp. $x_1$). However, it is common knowledge that the types are independent and the agents are efficient with probability $p$ and are inefficient $1-p$.

- After observing the terms of the contract, the agent either accepts or rejects the contract. In the latter case the game ends. If the agent accepts the contract, he discovers his type costless. Then he chooses the action that he prefers according to the terms of contract.

- In next period, agent works and the principal receives profit if the fraction of the agents fulfills the contract.

To cover broader cases of inequity averse preferences, we follow Fehr and Schmidt (1999)'s theory of fairness to extend our model by allowing agents who compare wages or rents. In order to apply their theory, we assume that (i) all agents are have identical and are inequity averse, (ii) employed agents compare themselves exclusively with other employed agents, (iii) we focus on envious motivation dominates altruistic one.

Following Siemens (2004), the inequity aversion under the adverse selection is that following additional notation and definition are helpful in order to provide a formal statement in our model. In all definitions below consider an agent of type $t_i$ with $\hat{t}_k$ indicating that he has announced to be of type $t_k$. Suppose the agent is employed and has to satisfy the menu of contract $\{(w_0, x_0), (w_1, x_1)\}$. Let

$$R_i(t_i, \hat{t}_k) = w_i - t_i x_i - \alpha \sum_{j=0,1} p_j \max[w_j - w_k, 0]$$

denote the agent's overall utility as his wage minus both the cost function and agent's concern for inequity of comparing wage. Note that the term $\alpha \sum_{j=0,1} p_j \max[w_j - w_k, 0]$ specifies the concern for inequity where $\alpha \geq 0$ is the measure of inequity and $p_j$ is the fraction of agents reporting to be of type $t_j$. On the one hand, according to inequity averse agent exclusively compare their own wage with wages of the other agents. On the other hand, the second definition of inequity aversion perfectly incorporates cost function. Let

$$R_i(t_i, \hat{t}_k) = w_i - t_i x_i - \alpha \sum_{m=0,1} \sum_{j=0,1} p_{jm} \max[w_m - t_j x_m - (w_i - t_k x_i), 0]$$

denote the agent's overall utility as his wage minus both the cost function and agent's concern for inequity of comparing rent. Note that the term $p_{jm}$ is the fraction of agents of type $t_m$ reporting to be of type $t_j$.

Given the definition of overall utility, we assume that $\max[0,0]=0$, and the domain of $R(\cdot)$ is $(-\infty,+\infty)$, that $\lim_{w\to-\infty}\max[\cdot]=-\infty$ and that $\lim_{w\to\infty}\max[\cdot]=\overline{\max}[\cdot]$, where $\overline{\max}[\cdot]$ may be infinite[1].

# 3 Results of Adverse Selection under Inequity Aversion

## 3.1 Comparing Wages of Risk-Averse Agents

Due to $t$ only taking two values, the difference $t_1-t_0$ is denoted $\Delta t$ in below all section. Suppose that the principal wants to employ both types of agents and the production cost of the agent is unobservable. Given that the contract between the principal and the agent is signed before the agent discovers his type, agent's ex ante participation constraint is written as

$$pu(w_0-t_0x_0)+(1-p)u(w_1-t_1x_1)\geq 0 \tag{PC}$$

which the contract must guarantee agent's reservation utility, taken to be zero. For simplicity, we use the notation $U_0=w_0-t_0x_0$ and $U_1=w_1-t_1x_1$ to denote the respective information rent of each type. Given a direct revelation mechanism, the principal's program $\mathbf{P}$ is given by

$$\max_{U_i,x_i,i=0,1} p[S(x_0)-t_0x_0-U_0]+(1-p)[S(x_1)-t_1x_1-U_1] \tag{$\mathbf{P}$}$$

subject to

$$pu(U_0)+(1-p)u(U_1)\geq 0 \tag{PC}$$

$$U_0\geq U_1+\Delta tx_1-\alpha\sum_{j=0,1}p_j\max[w_j-w_1,0] \tag{ICG}$$

$$U_1\geq U_0-\Delta tx_0+\alpha\sum_{j=0,1}p_j\max[w_j-w_1,0] \tag{ICB}$$

[1] The features of this specification are the applicability of the standard separability axiom for inequity aversion preferences. See Neilson (2003). Itoh (2004) analyzes how inequity aversion among agents changes optimal incentive contracts assuming limited liability to be the source of moral hazard.

The incentive compatibility constraint (ICG) ensures that the efficient agent will not gain by announcing $t_1$. The incentive compatibility constraint (ICB) is that the inefficient agent truthfully reports his private information, but as is usual in adverse selection problems, this constraint is not binding, and we can omit it.

To distinguish notation, the production level $x_i$ and information rent $U_i, i = 0, 1$ are defined as the optimal production $x_i^\star$, and the optimal ex post information rents are denoted as $U_i^\star$ for agents. The solution of above program **P** leads the following result:

**Proposition 1** *Suppose that agents compare wages. When the agent is risk-averse and inequity averse and contracting takes place ex ante, the optimal menu of contracts entails:*
*(i) A upward output distortion for the efficient type* $x_0^\star > x_0^{FB}$, *with*

$$S^{'}(x_0^\star) = t_0 - t_0\alpha\frac{p(1-p)[u^{'}(U_1) - u^{'}(U_0)]}{(1+\alpha p)[pu^{'}(U_0) + (1-p)u^{'}(U_1)]} < S^{'}(x_0^{FB}) = t_0, \tag{1}$$

*where* $x_0^{FB}$ *is the first-best level of production.*
*(ii) A downward output distortion for the inefficient type* $x_1^\star < x_1^{SB}$, *with*

$$\begin{aligned} S^{'}(x_1^\star) &= t_1 + \frac{p[u^{'}(U_1) - u^{'}(U_0)]}{(1+\alpha p)[pu^{'}(U_0) + (1-p)u^{'}(U_1)]}\Delta t \\ &+ \alpha t_1\frac{p^2[u^{'}(U_1) - u^{'}(U_0)]}{(1+\alpha p)[pu^{'}(U_0) + (1-p)u^{'}(U_1)]} \\ &> S^{'}(x_1^{SB}) = t_1 + \frac{p[u^{'}(U_1) - u^{'}(U_0)]}{[pu^{'}(U_0) + (1-p)u^{'}(U_1)]}\Delta t, \end{aligned} \tag{2}$$

*where* $x_1^{SB}$ *is defined by second-best solution of the standard adverse selection.*
*(iii) monotonicity constraint holds;* $x_0^\star > x_1^\star$.
*(iv) Both (PC) and (ICG) are the only binding constraints. The efficient (resp. inefficient) type gets a strictly positive (resp. negative) ex post information rent,* $U_0^\star > 0 > U_1^\star$.

**Proof**: We can now substitute for the information rent and write the Lagrangian of the principal's program

$$L = p[S(x_0) - t_0x_0 - U_0] + (1-p)[S(x_1) - t_1x_1 - U_1]$$
$$+\lambda[U_0 - U_1 - \Delta t x_1 + \alpha p(U_0 + t_0x_0 - U_1 - t_1x_1)] + \pi[pu(U_0) + (1-p)u(U_1)],$$

where $\lambda$ is the multiplier of (ICG) and $\pi$ is the multiplier of (PC).

The first-order conditions of this program can be expressed respectively as

$$\frac{\partial L}{\partial U_0} = -p + \lambda + \lambda\alpha p + \pi p u^{'}(U_0) = 0 \tag{3}$$
$$\frac{\partial L}{\partial U_1} = -(1-p) - \lambda - \lambda\alpha p + \pi(1-p)u^{'}(U_1) = 0 \tag{4}$$
$$\frac{\partial L}{\partial x_0} = pS^{'}(x_0^{\star}) - pt_0 + \lambda\alpha p t_0 = 0 \tag{5}$$
$$\frac{\partial L}{\partial x_1} = (1-p)S^{'}(x_1^{\star}) - (1-p)t_1 - \lambda\Delta t - \lambda\alpha p t_1 = 0 \tag{6}$$

Summing (3) and (4), we obtain

$$\pi[pu^{'}(U_0) + (1-p)u^{'}(U_1)] = 1 \tag{7}$$

and thus $\pi > 0$. Using (7) and inserting it into (3) yields

$$\lambda = \frac{p(1-p)[u^{'}(U_1) - u^{'}(U_0)]}{(1+\alpha p)[pu^{'}(U_0) + (1-p)u^{'}(U_1)]} \tag{8}$$

Moreover, (ICG) implies that $U_0^{\star} \geq U_1^{\star}$ and thus $\lambda \geq 0$, with $\lambda > 0$ for a positive output $x_0$.

Simplifying by using (5), (6) and (8) yields (1) and (2) in the proposition 1, respectively.

In order to show $x_1^{\star} < x_1^{SB}$, it is sufficient to show that $S^{'}(x_1^{\star}) > S^{'}(x_1^{SB})$. To compare the optimal output levels, let be $A \equiv [u^{'}(U_1) - u^{'}(U_0)]$ and $B \equiv [pu^{'}(U_0) + (1-p)u^{'}(U_1)]$. Thus, we can check that

$$\begin{aligned} S^{'}(x_1^{\star}) > S^{'}(x_1^{SB}) &\Leftrightarrow \frac{pA}{(1+\alpha p)B}\Delta t + \alpha t_1 \frac{p^2 A}{(1+\alpha p)B} > \frac{pA}{B}\Delta t \\ &\Leftrightarrow \alpha p t_1 > \alpha p \Delta t \\ &\Leftrightarrow t_1 > t_1 - t_0 \Leftrightarrow x_1^{\star} < x_1^{SB}. \end{aligned}$$

In order to show $x_0^{\star} > x_1^{\star}$, it is sufficient to show that $S^{'}(x_1^{\star}) > S^{'}(x_0^{\star})$. By comparing the optimal output levels given by (1) and (2), we can easily check.

Using binding (ICG) and inserting it into (ICB) yields

$$0 > -\Delta t(x_0^{\star} - x_1^{\star}). \tag{9}$$

Solving binding (ICG) immediately yields,

$$U_0^\star + U_1^\star = \frac{\Delta t x_1^\star - \alpha p(t_0 x_0^\star - t_1 x_1^\star)}{1+\alpha p}. \tag{10}$$

Since $S^{'}(x_0^\star)$ is decreasing in $\alpha$, the efficient type must have a positive rent and be $\frac{t_0}{t_1} < \frac{x_1^\star}{x_0^\star}$. Otherwise $S^{'}(x_0^\star)$ is increasing in $\alpha$ as long as ex ante participation constraint is binding.

Q.E.D.

Proposition 1 says that the difference between $U_0^\star$ and $U_1^\star$ makes the inefficient risk-averse agent with inequity aversion bear less risk than the absence of concerns with inequity of adverse selection problem. On the other hand, the efficient agent bears more risk than the standard solution of adverse selection. To guarantee the participation of inequity and risk-averse agent, the principal must pay not only a risk premium but also an inequity premium. Reducing these premiums call for the downward in the inefficient type's output and the upward distortion in the efficient type's output. Hence risk-aversion in presence of concern with inequity leads the principal to weaken incentive for inefficient agents, and to strengthen incentive for efficient agents if they compare wages among themselves[2].

Inspection of $x_0^\star$ supports that the efficient agents' output is strictly positive for all $\alpha$. Since $S^{'}(x_0)$ is decreasing whereas $S^{'}(x_1)$ is increasing in $\alpha$, the efficient type's rent must be $\frac{t_0}{t_1} < \frac{x_1}{x_0}$ and increase in $\alpha$. Otherwise $S^{'}(x_0)$ is increasing in $\alpha$. In particular, inequity aversion does influence agents' ex post information rents. Thus, the difference of ex post information rent diverges as $\alpha$ is higher.

Increasing $\alpha$ induces the efficient agent's output to be high and makes the inefficient agent's output be small. Hence, there exists a cutoff level $\hat{\alpha}$ so that, for all $\alpha > \hat{\alpha}$, $\frac{t_0}{t_1}$ is equal to $\frac{x_1}{x_0}$ which means that $t_0 x_0 = t_1 x_1$. As both types can be indifferent between two contracts which equal wage levels if cost functions are identical, there is no inequity where ex post information rents are equal to $U_0^\star = U_1^\star = 0$. Summarizing the above explanations yields the following result.

[2]Setting $\alpha = 0$ yields the standard solution in the adverse selection framework, where the agent perfectly has no envy.

**Proposition 2** *Suppose that agents compare wages. When the agent is risk-averse and inequity averse and contracting takes place ex ante, the optimal menu of contracts entails:*
*(i) If $\alpha \leq \hat{\alpha}$, the difference of ex post information rents diverges as $\alpha$ becomes high.*
*(ii) If $\alpha > \hat{\alpha}$, both types of agents' incentive constraint is binding, with $U_1^\star = U_0^\star = 0$.*

Consider an increase of the efficient agents' output with increasing $\alpha$. As efficient agents incur higher production costs they must be received a higher wage. This increases the difference of ex post information rent for which the inefficient agents must be compensated from the second best level of output $x_1^{SB}$ to $x_1^\star$, and their cost must increase by more than their ex post information rent. However, this inequity aversion makes the principal by more ex post information rent for the efficient agent comparatively cheaper if and only if inefficient agents envy below the cutoff level of the degree of inequity. It is the reason which becomes larger with more having made production of an efficient agent increase and paying ex post information rent.

From the proposition 2, as agent's punishment becomes high, other agent's ex post information rent is also higher. However, comparing ex post information rents with standard adverse selection's provides following lemma 1.

**Lemma 1**: *Suppose that (ICB) is binding and agents compare wages. The optimal difference in the model $\Delta U^\star \equiv U_0^\star + U_1^\star$ is always smaller than $\Delta U^S \equiv U_0^S + U_1^S$ in the standard adverse selection problem.*

**Proof**: Using $x_0^\star > x_0^{FB} > x_1^{SB} > x_1^\star$ and (ICG) with binding, we obtain

$$\Delta U^\star = \frac{\Delta t x_1^\star - \alpha p(t_0 x_0^\star - t_1 x_1^\star)}{1+\alpha p} < \Delta U^S \equiv \Delta t x_1^{SB}$$
$$\Leftrightarrow -\alpha p[t_1(x_1^{SB} - x_1^\star) + t_0(x_0^\star - x_1^{SB})] - (1+\alpha p)[\Delta t(x_0^\star - x_1^\star) + 2\Delta t(x_0^{FB} - x_1^{SB}) + U_0^\star]$$
$$< (t_1 - t_0)(x_1^{SB} - x_1^\star).$$

Q.E.D.

Throughout this subsection suppose workers compare wages. Consider first the optimal employment contracts if the firm has to employ both types of workers. Due to the maximum functions in the workers' suffering their utility functions have a kink at $w_0 = w_1$ and are thus not differentiable at this point. This can cause difficulties in the maximization problem, but fortunately incentive compatibility implies the following lemma.

**Lemma 2**: *Suppose that agents compare wages, and the firm employs both types of workers. Then* $w_0 \geq w_1$ *for* $\alpha \geq 0$

**Proof**: Suppose $w_0 < w_1$ for some $\alpha$ so that the rent is given by $R_0(t_0, \hat{t}_0) = U_0 - (1 - p)\alpha(U_0 + t_0 x_0 - U_1 - t_1 x_1)$ and $R_1(t_1, \hat{t}_1) = U_1$. As (PC) is binding the optimal incentive contracts must satisfy the following remaining constraints

$$(w_0 - w_1)[1 + \alpha(1-p)] + t_0(x_1^\star - x_0^\star) = 0 \qquad \text{(ICG}^{'}\text{)}$$

$$(w_1 - w_0)[1 + \alpha(1-p)] - t_1(x_1^\star - x_0^\star) > 0 \qquad \text{(ICB}^{'}\text{)}$$

After substitution of the above expression (ICB$^{'}$) is satisfied if and only if $(t_0 - t_1)(x_1^\star - x_0^\star) > 0$. As $t_0 < t_1$ this contradicts from (ICG$^{'}$) since $S^{'}(0) = +\infty$ and monotonicity constraints on $x_0^\star$ and $x_1^\star$ are violated. Q.E.D.

Lemma 2 means that the efficient agents are weakly preferred inefficient agents' contract to their own when agent envy not large. Hence, the considered contract can be incentive compatible.

### 3.2 Comparing Rents of Risk-Averse Agents

From the result of proposition 1 with binding (ICG), when agents compare rents, it is easily checked that $U_0(t_0, \hat{t}_0) < U_1(t_1, \hat{t}_1)$ cannot be the incentive compatible in the sense that the inefficient agent never get rent[3]. Thus asymmetric information and screening cause a rent inequality. In other words, the efficient agents get a weakly higher rent so that only inefficient agents suffer from inequity and therefore $R_1(t_1, \hat{t}_1) = U_1 - \alpha \sum_{m=0,1} \sum_{j=0,1} p_{jm} \max[w_m - t_j x_m - (w_1 - t_1 x_1), 0]$ and $R_0(t_0, \hat{t}_0) = U_0$ when agents

[3]See Siemens (2004)'s lemma 4.

compare rents. Since $S^{'}(0) = +\infty$ and $\lim_{x\to 0} S^{'}(x) = 0$, the inefficient agent's production is strictly positive so that the principal's maximization program is applicable. Hence the principal's program **Pr** is given by

$$\max_{U_0,x_0,U_1,x_1} p[S(x_0) - t_0x_0 - U_0] + (1-p)[S(x_1) - t_1x_1 - U_1] \qquad \textbf{Pr}$$

subject to

$$pu(U_0) + (1-p)u(U_1) \geq 0 \qquad \text{(PC)}$$

$$U_0 \geq U_1 + \Delta t x_1 \qquad \text{(ICGr)}$$

$$U_1 \geq U_0 - \Delta t x_0 + \alpha \sum_{m=0,1} \sum_{j=0,1} p_{jm} \max[w_m - t_j x_m - (w_i - t_k x_i), 0] \qquad \text{(ICBr)}$$

Second constraint (ICGr) is an incentive compatibility constraint which state that the overall utility of the efficient agent is always larger than that of the inefficient when agents compare rents. Therefore, (ICBr) becomes the incentive compatibility constraint as for which only inefficient agent feels inequity for rents. Given the principal's program **Pr**, we summarize the next proposition.

**Proposition 3** *Suppose that agents compare rents. When the agent is risk-averse and inequity averse and contracting takes place ex ante, the optimal menu of contracts entails: (i) No output distortion for the efficient type* $x_0^\star = x_0^{FB}$ *and a downward distortion for the inefficient type* $x_1^\star = x_1^{SB}$*, with*

$$S^{'}(x_1^\star) = t_1 + \frac{p[u^{'}(U_1) - u^{'}(U_0)]}{[pu^{'}(U_0) + (1-p)u^{'}(U_1)]}\Delta t = S^{'}(x_1^{SB}) \Leftrightarrow x_0^\star = x_0^{FB} > x_1^\star = x_1^{SB}.$$

*(ii) Both (PC) and (ICGr) are the only binding constraints if and only if* $x_1^\star < \frac{x_0^\star}{1+\alpha p}$*. The efficient (resp. inefficient) type gets a strictly positive (resp. negative) ex post information rent,* $U_0^\star > 0 > U_1^\star$ *if and only if* $x_1^\star < \frac{x_0^\star}{1+\alpha p}$*.*

**Proof**: The details of the proof can be omitted, as they parallel the analysis in the proofs of proposition 1. Using (ICGr) with binding and inserting it into (ICBr) yields

$$0 \geq \Delta t[x_1^\star(1+\alpha p) - x_0^\star)]$$

if $x_1^\star \leq \frac{x_0^\star}{1+\alpha p}$ as long as ex ante participation constraint is binding. Otherwise a menu of contracts is not incentive feasible which the revelation mechanism should not hold. Q.E.D.

Even if the contract is signed before the risk-averse agent with inequity aversion discovers his type, optimal outputs in this model are the same as with traditional adverse selection framework of ex ante participation constraint. But ex post information rents differ from those analyzed by standard solutions of adverse selection and Siemens (2004).

In particular, inequity aversion does influence inefficient agent's ex post information rent. If $x_1 \leq \frac{x_0}{1+\alpha p}$, then there exists cutoff level $\hat{\alpha}$ so that for all $\alpha \geq \hat{\alpha}$ the (ICBr) is binding. As both types of agents receive the same wage if cost functions are identical, there is no inequity where ex post information rents are equal to $U_0^\star = U_1^\star = 0$. This provides the following proposition.

**Proposition 4** *Suppose that agents compare rents.*
*(i) If $x_1^\star \leq \frac{x_0^\star}{1+\alpha p}$, there exists a cutoff $\hat{\alpha}$ so that for all $\alpha < \hat{\alpha}$ the inefficient agent's incentive compatibility constraint is non-binding and the difference of ex post information rent converges as $\alpha$ becomes high.*
*(ii) If $\alpha \geq \hat{\alpha}$, the ex post information rents are equal to $U_0^\star = U_1^\star = 0$.*

In contrary to Siemens (2004), the difference of ex post information rent decreases as inequity concerns for all $\alpha < \hat{\alpha}$ when agents compare rents among themselves.

Additionally, for the case when there is no inequity ($\alpha = 0$) of standard adverse selection, we get the result of comparison as lemma.

**Lemma 3**: *Suppose that (ICBr) is non-binding. The optimal difference $\Delta U^\star$ is always larger than the optimal difference $\Delta U^S$ if agents compare rents.*

**Proof**: Using $x_0^\star = x_0^{FB} > x_1^\star = x_1^{SB}$ and simplifying by (ICGr) with binding, we also obtain

$$\Delta U^\star > \Delta U^S \Leftrightarrow \Delta t x_1^\star + \Delta t[x_1^\star(1+\alpha p) - x_0^\star] > \Delta t x_1^\star - \Delta t(x_0^\star - x_1^\star).$$
$$\Leftrightarrow x_1^\star \alpha p > 0. \qquad \text{Q.E.D.}$$

If the agent is risk neutral with inequity aversion and compares rents, his ex ante participation constraint is now written as

$$pU_0 + (1-p)U_1 \geq 0.$$

This ex ante participation constraint replaces the two interim participation constraints in comparing rents' problem **Pr**. We have the following characterization of the optimal contract.

**Corollary**: *Suppose that the agent compare rents. Then when the agent is risk-neutral with inequity aversion and contracting takes place ex ante the optimal incentive contract implements the first-best output with* $U_0^\star = (1-p)\Delta t x_1$ *and* $U_1^\star = -p\Delta t x_1$.

**Proof**: The solution is the same as with risk neutrality in Laffont and Martimort (2002, pp. 57-58) with absence of concern with inequity. Q.E.D.

## 4 Conclusions and Applications

We have found that the efficient agent who is a risk-averse in presence of concerns with inequity produces more than first-best level, whereas the inefficient agent produces less than second-best level. Hence risk-aversion with inequity leads the principal to weaken incentive for inefficient agents, and to strengthen for efficient agents if they compare wages and envy is not large. On the other hand, when risk-averse agents compare their information rents among themselves, optimal outputs in this model are the same as with traditional adverse selection framework of ex ante participation constraint. But ex post information rents differ from those analyzed by standard solutions of adverse selection.

Due to introducing concerns with inequity into adverse selection, optimal incentive contract structures often differ from those predicted by standard solutions canonical adverse selection problem. The precise details of optimal contract structure in the presence of aversion to inequity await future research. The extent to which introducing cost of gathering information of types before contracting will alter results could be explored.

Let us point to possible applications that fit into the model presented here. As already mentioned, if the agents are risk-averse with inequity aversion and they compare wages, the model endogenously generates $x_0^\star > x_0^{FB} > x_1^{SB} > x_1^\star$, which the efficient (resp. inefficient) agent produces more (resp. less) than under the standard adverse selection framework. It is applicable that consumers (agents) buy one unit of a commodity with quality $t_i$ but are vertically differentiated with respect to their preferences for the good and the principal is a firm which faces a continuum of consumer with utility $\tilde{U} = tu(x) - w$. The marginal cost of producing one unit of quality $x$ is $c(x)$ and the firm has the utility function $S(w) - c(x)$.

Suppose that the firm faces two possible consumer groups who are comparing qualities of brand productions with those of others. Before they know their preferences, the following conclusion would emerge: (i) If the consumer groups compare qualities of brand productions with those of others, the firm has an incentive to produce qualities between more reduced below the second-best for the low valuation agent and above the first best for the high valuation agent when agents have relatively small envy among them (e.g., qualities by more broader vertical differentiation). Contrary to standard adverse selection framework, this is due to the fact that there is not only the high valuation of the informed consumer group who has a more high willingness to pay but also the low valuation of the informed consumer group who has a less low willingness to pay. (ii) If the consumer groups compare rents with those of others, there exits a downward distortion of the low valuation group's quality, as is in standard adverse selection. However, as envy increased, broader vertical differentiation will be provided beacuse the difference of ex post information rents is reduced. This is due to the fact that there is the low valuation of the informed consumer group who has less willingness to pay than that of the high valuation but more higher punishment is provided by the firm.

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